



Georgia Standards of Excellence Frameworks

Mathematics

GSE Grade 6

Unit 2: Rate, Ratio, and Proportional
Reasoning Using Equivalent Fractions



Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 2

Rate, Ratio, and Proportional Reasoning Using Equivalent Fractions

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***Revised standards, new tasks, and other new resources indicated in bold red font.**

** Existing task modified in the July 2015 edition

OVERVIEW

In this unit, students will:

- gain a deeper understanding of proportional reasoning through instruction and practice
- develop and use multiplicative thinking
- develop a sense of proportional reasoning
- develop the understanding that ratio is a comparison of two numbers or quantities
- find percents using the same processes for solving rates and proportions
- solve real-life problems involving measurement units that need to be converted

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
- 2. Reason abstractly and quantitatively.** Students understand the relationship between two quantities in order to express them mathematically. They use ratio and rate notation as well as visual models and contexts to demonstrate reasoning.
- 3. Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding appropriateness of representations given ratio and rate contexts. For example, does a tape diagram adequately represent a given ratio scenario.
- 4. Model with mathematics.** Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.
- 5. Use appropriate tools strategically.** Students choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.
- 6. Attend to precision.** Students use and interpret mathematical language to make sense of ratios and rates.
- 7. Look for and make use of structure.** The structure of a ratio is unique and can be used across a wide variety of problem-solving situations. For instance, students recognize patterns that exist in ratio tables, including both the additive and multiplicative properties. In addition, students use their knowledge of the structures of word problems to make sense of real-world problems.
- 8. Look for and express regularity in repeated reasoning.** Students utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts. Students can generalize the relationship between representations, understanding that all formats represent the same ratio or rate.

STANDARDS FOR MATHEMATICAL CONTENT

Understand ratio concepts and use ratio reasoning to solve problems.

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

MGSE6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. For example, given $1 \text{ in.} = 2.54 \text{ cm}$, how many centimeters are in 6 inches?

BIG IDEAS

- A ratio is a number that relates two quantities or measures within a given situation in a multiplicative relationship (in contrast to a difference or additive relationship). The relationships and rules that govern whole numbers, govern all rational numbers.
- Making explicit the type of relationships that exist between two values will minimize confusion between multiplicative and additive situations.
- Ratios can express comparisons of a part to whole, (a/b with $b \neq 0$), for example, the ratio of the number of boys in a class to the number of students in the class.
- The ratio of the length to the width of a rectangle is a part-to-part relationship.
- Understand that fractions are also part-whole ratios, meaning fractions are also ratios. Percentages are ratios and are sometimes used to express ratios.

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- Both part-to-whole and part-to-part ratios compare two measures of the same type of thing. A ratio can also be a rate.
- A rate is a comparison of the measures of two different things or quantities; the measuring unit is different for each value. For example if 4 similar vans carry 36 passengers, then the comparison of 4 vans to 36 passengers is a ratio.
- All rates of speed are ratios that compare distance to time, such as driving at 45 miles per hour or jogging at 7 minutes per mile.
- Ratios use division to represent relations between two quantities.

ESSENTIAL QUESTIONS

- What kinds of problems can I solve by using ratios?
- How can I tell if a relationship is multiplicative?
- What is the difference between a multiplicative and an additive relationship?
- What are equivalent ratios?
- What are rates?
- How are unit rates helpful in solving real-world problems?
- How are ratios and rates similar and different?
- What are percentages?
- What information do I get when I compare two numbers using a ratio?

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

CONCEPTS & SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- multiples and factors
- divisibility rules
- relationships and rules for multiplication and division of whole numbers as they apply to decimal fractions
- understanding of common fractions

***FLUENCY**

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: <http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf> and: <http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/>

STRATEGIES FOR TEACHING AND LEARNING

Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Grade 6 is the first of several years in which students develop this multiplicative/proportional thinking. Examples with ratio and proportion should involve measurements, prices and geometric contexts. Miles per hour, constant speed, and portions per person within contexts that are relevant to sixth graders should be used for rates. Experience with proportional and non-proportional relationships, comparing and predicting ratios, and relating unit rates to previously learned unit fractions will facilitate the development of proportional reasoning. **Although algorithms provide efficient means for finding solutions, the cross-product algorithm commonly used for solving proportions will not aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage**

thinking about multiplicative situations instead of indiscriminately applying rules. Students will use the cross-product algorithm in unit 4 after solving one-step equations.

Students develop the understanding that ratio is a comparison of two numbers or quantities. Ratios that are written as part-to-whole are comparing a specific part to the whole. Fractions and percent are examples of part-to-whole ratios. Fractions are written as the part being identified compared to the whole amount. A percent is the part identified compared to the whole (100). Provide students with multiple examples of ratios, fractions and percent of this type. For example, the number of girls in the class (12) to the number of students in the class (28) is the ratio 12 to 28.

Percents are often taught in relationship to learning fractions and decimals. This cluster indicates that percents are to be taught as a special type of rate. Provide students with opportunities to find percents in the same ways they would solve rates and proportions.

Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16. This form of ratios is often used to compare the event that can happen to the event that cannot happen. Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon and students per bus. For example, 3 cans of pudding cost \$2.48 at Store A and 6 cans of the same pudding costs \$4.50 at Store B. Which store has the better buy on these cans of pudding? **Various strategies could be used to solve this problem:**

- **A student can determine the unit cost of 1 can of pudding at each store and compare.**
- **A student can determine the cost of 6 cans of pudding at Store A by doubling \$2.48.**
- **A student can determine the cost of 3 cans of pudding at Store B by taking $\frac{1}{2}$ of \$4.50.**

Using ratio tables develops the concept of proportion. By comparing equivalent ratios, the concept of proportional thinking is developed and many problems can be easily solved.

Store A	
3 cans	6 cans
\$2.48	\$4.96

Store B	
6 cans	3 cans
\$4.50	\$2.25

Students should also solve real-life problems involving measurement units that need to be converted. **Representing these measurement conversions with models such as ratio tables, t-charts or double number line diagrams will help students internalize the size relationships between same system measurements and relate the process of converting to the solution of a ratio.**

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Multiplicative reasoning is used when finding the missing element in a proportion. For example, use 2 cups of syrup to 5 cups of water to make fruit punch. If 6 cups of syrup are used to make punch, how many cups of water are needed?

$\frac{2}{5} = \frac{6}{x}$ Recognize that the relationship between 2 and 6 is 3 times. $(2)(3) = 6$

To find x, the relationship between 5 and x must also be 3 times. $(3)(5) = x$

$\frac{2}{5} = \frac{6}{15}$ the final proportion

Other ways to illustrate ratios that will help students see the relationships follow. Begin written representation of ratios with the words “out of” or “to” before using the symbolic notation of the colon and then the fraction bar; for example, 3 out of 7, 3 to 7, 3:7 and then $\frac{3}{7}$.

Use skip counting as a technique to determine if ratios are equal.

Labeling units helps students organize the quantities when writing proportions.

$$\frac{3\text{eggs}}{2\text{cupsofFlour}} = \frac{z\text{eggs}}{8\text{cupsofFlour}}$$

Using hue/color intensity is a visual way to examine ratios of part-to-part. Students can compare the intensity of the color green and relate that to the ratio of colors used. For example, have students mix green paint into white paint in the following ratios: 1 part green to 5 parts white, 2 parts green to 3 parts white, and 3 parts green to 7 parts white. Compare the green color intensity with their ratios.

MISCONCEPTIONS

Regarding proportional reasoning, the most common misconception that students have about proportional relationships is that they are additive rather than multiplicative. For example, if a particular shade of orange paint uses 3 quarts of red paint for every 2 quarts of yellow paint, a student using additive reasoning might incorrectly reason that 9 quarts of red paint should be mixed with 8 quarts of yellow paint to maintain the same shade of orange because “the amount of red paint should be 1 more quart than the amount of yellow paint.”

Regarding percents, often there is a misunderstanding that a percent is always a natural number less than or equal to 100. Provide examples of percent amounts that are greater than 100%, and percent amounts that are less 1%.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

<http://www.amathsdictionaryforkids.com/>

This web site has activities to help students more fully understand and retain new vocabulary

<http://intermath.coe.uga.edu/dictionary/homepg.asp>

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

<http://www.corestandards.org/Math/Content/mathematics-glossary/glossary>

- **Percent:** A fraction or ratio in which the denominator is 100. A number compared to 100.
- **Proportion:** An equation which states that two ratios are equal.
- **Rate:** A comparison of two quantities that have different units of measure
- **Ratio:** compares quantities that share a fixed, multiplicative relationship.
- **Rational number:** A number that can be written as a/b where a and b are integers, but b is not equal to 0.
- **Tape diagram:** A thinking tool used to visually represent a mathematical problem and transform the words into an appropriate numerical operation. Tape diagrams are linear drawings that look like a segment of tape, used to illustrate number relationships. Also known as Singapore Strips, strip diagrams, bar models or graphs, fraction strips, or length models.

- **Unit Ratio:** are ratios written as some number to 1.
- **Quantity:** is an amount that can be counted or measured.

INSTRUCTIONAL RESOURCES AND TOOLS

- 100 grids (10 x 10) for modeling percent
- Bar Models – for example, 4 red bars to 6 blue bars as a visual representation of a ratio and then expand the number of bars to show other equivalent ratios
- Cuisenaire Rods
- Double Number Lines
- Ratio tables – to use for proportional reasoning
- Tape Diagrams

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students’ understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students’ understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student’s mathematical reasoning forward. More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

For middle and high schools, each Georgia Standards of Excellence mathematics unit includes at least one Spotlight Task. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

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Task	Task Type Grouping Strategy	Content Addressed	Standards Addressed
*<u>Arcade Basketball Insanity</u>	Constructing Task <i>Partner/Small Group Task</i>	Ratios, Rates, and Unit Rates	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3b
<u>Rope Jumper</u> (Spotlight Task)	Constructing Task <i>Partner/Small Group Task</i>	Ratios, Rates, and Unit Rates	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3b
<u>Snack Mix</u> (Spotlight Task)	Constructing Task <i>Partner/Small Group Task</i>	Ratios, Rates, and Unit Rates	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3b
<u>Real??? World Ratios</u> (Spotlight Task)	Constructing Task <i>Partner/Small Group Task</i>	Ratios, Rates, and Unit Rates	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3b
<u>Ratios and Rates</u>	Learning Task <i>Partner/Small Group Task</i>	Ratios, Rates, and Unit Rates	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3b
<u>Constant Dimensions</u>	Performance Task <i>Partner/Small Group Task</i>	Ratio Units	MGSE6.RP.1 MGSE6.RP.3 MGSE6.RP.3a
**<u>How Many Noses Are In Your Arms?</u>	Performance Task <i>Partner/Partner Group</i>	Ratios and Proportions	MGSE6.RP.1 MGSE6.RP.3 MGSE6.RP.3d
<u>Reaching the Goal</u>	Performance Task <i>Individual/Partner Task</i>	Fraction, Percent	MGSE6.RP.3 MGSE6.RP.3c
*<u>Free Throw Warm-up</u>	Learning Task <i>Partner/Small Group Task</i>	Fractions, Decimals, Percents, Ratios, Rates	MGSE6.RP.3 MGSE6.RP.3c
<u>Free Throws</u>	Performance Task <i>Individual/Partner Task</i>	Multiplying, Percentages, and Ratios	MGSE6.RP.3 MGSE6.RP.3c
*<u>Comparing Rates</u>	Learning Task <i>Partner/Small Group Task</i>	Comparing Ratios and Rates	MGSE6.RP.1 MGSE6.RP.3 MGSE6.RP.3a

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<u>Traveling to School</u>	Formative Assessment Lesson	Fractions, ratios, and proportions	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3a MGSE6.RP.3b MGSE6.RP.3c MGSE6.RP.3d
<u>Optimizing Security Cameras</u>	Formative Assessment Lesson	Percentages	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3a MGSE6.RP.3b MGSE6.RP.3c MGSE6.RP.3d
<u>Ice Cream or Cake?</u>	Performance Task <i>Individual/Partner Task</i>	Percentages and Ratios	MGSE6.RP.3 MGSE6.RP.3c
<u>Culminating Task: The Rocky Mountain Vacation Trip Problem</u>	Performance Task <i>Individual</i>	Ratio and Proportions	MGSE6.RP.1 MGSE6.RP.2 MGSE6.RP.3 MGSE6.RP.3a MGSE6.RP.3b MGSE6.RP.3c MGSE6.RP.3d

* **Arcade Basketball Insanity**

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Task adapted from brianlack.wordpress.com

In this inquiry-based task, students will use proportional reasoning to predict the number of basketball shots that will be made in a given amount of time.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MGSE6.RP.3 **Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.**

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students make sense of ratio and unit rates in real-world contexts. They persevere by selecting and using appropriate representations for the given contexts.
- 2. Reason abstractly and quantitatively.** Students will reason about the value of the rational number in relation the models that are created to represent them.
- 3. Construct viable arguments and critique the reasoning of others.** Students use arguments to justify their reasoning when creating and solving proportions used in real-world contexts.
- 4. Model with mathematics.** Students create models using tape diagrams, double number lines, manipulatives, tables and graphs to represent real-world and mathematical situations involving ratios and proportions. For example, students will examine the relationships between slopes of lines and ratio tables in the context of given situations.
- 5. Use appropriate tools strategically.** Students use estimation as a sense-making tool to help them monitor and evaluate the reasonableness of their solution(s).
- 6. Attend to precision.** Students attend to the ratio and rate language studied in grade 6 to represent and solve problems involving rates and ratios.
- 7. Look for and make use of structure.** Students look for patterns that exist in ratio tables in order to make conjectures about solving the problem presented in this task.
- 8. Look for and express regularity in repeated reasoning.** Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

ESSENTIAL QUESTIONS

- What conditions help to recognize and represent proportional relationships between quantities?
- How are proportional relationships used to solve multistep ratio problems?

MATERIALS REQUIRED

- Videos for Arcade Basketball Insanity: 3-Act task (linked below)
- Recording sheet (attached)

TIME NEEDED

- 1 day

TEACHER NOTES

In this task, students will watch the video and then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need—and ask for it—it will be given to them.

TASK DESCRIPTION

The following 3-Act Task can be found at: <http://wp.me/p4Arxs-7>

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:

Watch the video:

<http://swf.tubechop.com/tubechop.swf?vurl=QjPvCAx4zGA&start=3.83&end=10.11&cid=5598521>

Students are asked what they noticed in the video. Students record what they noticed or wondered on the recording sheet. You may need to replay the video several times for students to be able to create good, mathematically relevant observations and questions. Students are asked to discuss and share what they wondered (or are curious about) as related to what they saw in the video.

Important Note: Although the MAIN QUESTION of this lesson is “*How many shots will he (the man on the left) make before time expires?*” it is important for the teacher to not ignore student generated questions and try to answer as many of them as possible (before, during, and after the lesson as a follow up).

Main Question: *How many shots will he (the man on the left) make before time expires?*
Write down an estimate you know is too high, and one you know is too low.

ACT 2:

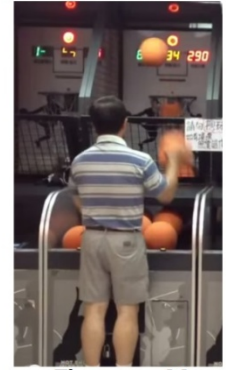
Students will realize that they do not have enough information to complete the problem. Release the following information to students ONLY AFTER they have identified what information they need.

Required information:

- The man on the left makes 15 shots in the first 6 seconds.
- There are 34 seconds remaining in the game.

Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem-solve. After individual time, students may work in groups (3-4 students) to collaborate and solve this task together.

He makes 15 shots in the first 6 seconds.



There are 34 seconds left.

ACT 3

Students will compare and share solution strategies.

- Allow students to share their solutions and reasoning.
- Show students the Act-3
 Video: <http://swf.tubechop.com/tubechop.swf?vurl=bpyX5E1tARk&start=3.62&end=53.85&cid=5598621>
 Reveal the answer (*He makes 85 shots in the last 34 seconds*).
 Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy. *Some students may realize that the man is averaging 5 shots made every 2 seconds; therefore, multiplying the shots made and the seconds by a factor of 17 yields a prediction of 85 shots made in the last 34 seconds. Others may use rate tables, tape diagrams, etc., as pictured below:*

shots made	15	30	45	60	75
seconds	6	12	18	24	30

The man should make 70 shots in 30 seconds. Since 4 seconds is 2/3 of 6 seconds, then 10 is 2/3 of 15 shots. Therefore, the man should make 75 + 10 (or 85 shots) in the last 34 seconds.

- Revisit any initial student questions that weren't answered.

Formative Assessment Questions

- How did the information that was revealed in Act 2 help you answer the question?
- What is the mathematical relationship between shots made and seconds?
- What models did you create?
- What organizational strategies did you use?

Arcade Basketball Insanity

Name: _____

Adapted from Andrew Stadel

ACT 1

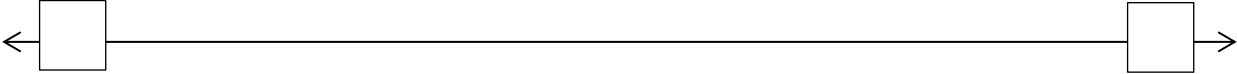
What did/do you notice?

What questions come to your mind?

Main Question: _____

Estimate the result of the main question. Explain.

Place an estimate that is too high and too low on the number line



Low estimate Place an "x" where your estimate belongs High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc...)

If possible, give a better estimate using this information: _____

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
<input type="checkbox"/> Make sense of problems & persevere in solving them	<input type="checkbox"/> Use appropriate tools strategically.
<input type="checkbox"/> Reason abstractly & quantitatively	<input type="checkbox"/> Attend to precision.
<input type="checkbox"/> Construct viable arguments & critique the reasoning of others.	<input type="checkbox"/> Look for and make use of structure.
<input type="checkbox"/> Model with mathematics.	<input type="checkbox"/> Look for and express regularity in repeated reasoning.

Rope Jumper (Spotlight Task)

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Task adapted from www.gfletchy.wordpress.com

In this inquiry-based task, students will use proportional reasoning to predict the number of times a professional jump roper will be able to jump a rope in a given amount of time.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MGSE6.RP.3 **Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.**

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of ratio and unit rates in real-world contexts. They persevere by selecting and using appropriate representations for the given contexts.

2. Reason abstractly and quantitatively. Students will reason about the value of the rational number in relation the models that are created to represent them.

3. Construct viable arguments and critique the reasoning of others. Students use arguments to justify their reasoning when creating and solving proportions used in real-world contexts.

4. Model with mathematics. Students create models using tape diagrams, double number lines, manipulatives, tables and graphs to represent real-world and mathematical situations involving ratios and proportions. For example, students will examine the relationships between slopes of lines and ratio tables in the context of given situations.

5. Use appropriate tools strategically. Students use visual representations such as the coordinate plane to show the constant of proportionality.

6. Attend to precision. Students attend to the ratio and rate language studied in grade 6 to represent and solve problems involving rates and ratios.

7. Look for and make use of structure. Students look for patterns that exist in ratio tables in order to make connections between the constant of proportionality in a table with the slope of a graph.

8. Look for and express regularity in repeated reasoning. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

ESSENTIAL QUESTIONS

- What conditions help to recognize and represent proportional relationships between quantities?
- How are proportional relationships used to solve multistep ratio and percent problems?
- How do equations represent proportional relationships? Explain how unit rates can be applied to model a real-world situation.

MATERIALS REQUIRED

- Videos for Rope Jumper – 3-Act task
- Recording sheet (attached)

TIME NEEDED

- 1-2 class periods

TEACHER NOTES

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them

TASK DESCRIPTION

The following 3-Act Task can be found at: <http://gfletchy3act.wordpress.com/rope-jumper/>

ACT 1:

Watch the video: <http://vimeo.com/93778595>

Ask students what they noticed and what they wonder. Record responses.
Suggested question: How many times will she be able to jump the rope in 30 seconds?
Estimate. Write an estimate that is too high and an estimate that is too low.

ACT 2:

The following information is provided for students as they ask for it.

- How many times she jumped the rope
- How long did she jump the rope for?

Watch the video: <http://vimeo.com/93778678>

ACT 3:

Watch the video: <http://vimeo.com/93777124>

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

Rope Jumper

Name: _____

Adapted from Andrew Stadel

ACT 1

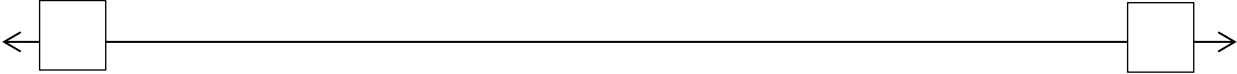
What did/do you notice?

What questions come to your mind?

Main Question: _____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line



Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc...)

If possible, give a better estimate using this information: _____

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
<input type="checkbox"/> Make sense of problems & persevere in solving them	<input type="checkbox"/> Use appropriate tools strategically.
<input type="checkbox"/> Reason abstractly & quantitatively	<input type="checkbox"/> Attend to precision.
<input type="checkbox"/> Construct viable arguments & critique the reasoning of others.	<input type="checkbox"/> Look for and make use of structure.
<input type="checkbox"/> Model with mathematics.	<input type="checkbox"/> Look for and express regularity in repeated reasoning.

Snack Mix (Spotlight Task)

[Back to Task Table](#)

Task adapted from <http://mikewiernicki.com/snack-mix>

In this inquiry-based task, students will use proportional reasoning to determine the amount of specific ingredients used in a snack recipe.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”*

MGSE6.RP.3 **Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.**

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students will make sense of the problem through its context before thinking about any numbers because only the context is evident from the video.
- 2. Reason abstractly and quantitatively.** Once students ask for quantities, they can begin to reason about the numbers in the context.
- 3. Construct viable arguments and critique the reasoning of others.**
- 4. Model with mathematics.** Students will use diagrams and or number lines to model the problem as well as equations.
- 6. Attend to precision.** Students will attend to precision using the language of mathematics in their discussions as well as in their calculations.
- 7. Look for and make use of structure.** Students will make use of structure by attending to the parts and the whole of the problem and understanding the relationships between them.

ESSENTIAL QUESTIONS

- How are unit rates helpful in solving real-world problems?

MATERIALS REQUIRED

- Video: Snack Mix
- 3-Act Recording Sheet (attached)

TIME NEEDED

- At least 2 days

TEACHER NOTES

In this task, students will watch the video and then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need—and ask for it—it will be given to them.

The 3-Act Task for this lesson can be found at: <http://mikewiernicki.com/snack-mix>

ACT 1:

Watch the video:

Ask students what they noticed. Record student responses on the board or on chart paper. Ask students what they wonder (or what they are curious about). Record student wonders on the board or on chart paper.

Suggested question: How much of each ingredient is needed to fill all of the cups?

Have students make an estimate of the number of ounces for each ingredient to make enough to fill all of the cups.

Have students make an estimate that is too low, and another estimate that is too high.

ACT 2:

Students should begin to work on solving the questions they are curious about. Students may have a lot of different solution pathways, but this is encouraged. Students should be working with a partner or a small group to bounce ideas off of. When students ask for a bit of information, give it to them. Below, you will find some information that students may ask for to solve this task.

ACT 3

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

ACT 4

DIFFERENTIATION

Extension:

Students may wish to continue with a solution for how much it would take to make this snack mix for their class or the entire 6th grade, or the entire middle school – including teachers, and administrators . . . this could be all the practice they need. Students may also wish to create their own snack recipe and find the ratios to make enough for the class.

Intervention:

Students needing support should be guided to use a tool such as counters and/or tiles (or other manipulatives) to represent the situation in the video. Students can then be asked to create equivalent ratios using these same materials. Students can then be asked to create a representation of this information using a tape diagram, double number line, or tables of equivalent ratios.

Snack Mix

Name: _____

Adapted from Andrew Stadel

ACT 1

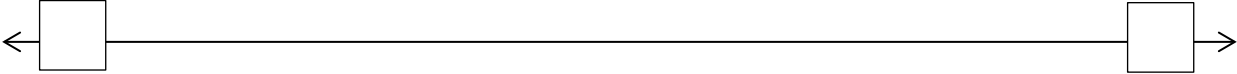
What did/do you notice?

What questions come to your mind?

Main Question: _____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line



Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc...)

If possible, give a better estimate using this information: _____

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
<input type="checkbox"/> Make sense of problems & persevere in solving them	<input type="checkbox"/> Use appropriate tools strategically.
<input type="checkbox"/> Reason abstractly & quantitatively	<input type="checkbox"/> Attend to precision.
<input type="checkbox"/> Construct viable arguments & critique the reasoning of others.	<input type="checkbox"/> Look for and make use of structure.
<input type="checkbox"/> Model with mathematics.	<input type="checkbox"/> Look for and express regularity in repeated reasoning.

Real??? World Ratios (Spotlight Task)

[Back to Task Table](#)

Adapted from www.illustrativemathematics.org

In this task, students will use express ratios as unit rates.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will use different contexts to write and make sense of ratios.

2. Reason abstractly and quantitatively. Students must reason abstractly and quantitatively in order to answer the questions because they can't rely on their experience with the situation to guide them through it.

3. Construct viable arguments and critique the reasoning of others. Students will share solutions and discuss reasonableness of these solutions within and between groups.

4. Model with mathematics. Students will use materials and/or diagrams to demonstrate their understanding of ratios.

6. Attend to precision. Students will attend to precision by using the language of mathematics in their discussions and explanations of their mathematical computations.

ESSENTIAL QUESTIONS

- What is a ratio?
- What are some ways I can think about ratios?
- How are fractions and ratios related?

MATERIALS REQUIRED

- Various manipulatives such as tiles and counters
- Real??? World Ratios task sheet (attached)

TIME NEEDED

- 2 days

TEACHER NOTES

Ratios use a multiplicative relationship to compare two quantities or measures. A ratio can relate one part of a whole to another part of the same whole (8 girls in a class to 7 boys in the same class). The ratio can be represented as $\frac{8}{7}$ or “a ratio of eight to seven” not “eight-sevenths” (the fraction). Ratios are often written with a fraction bar, so this distinction is necessary for students. This distinction should not be memorized. Ratios should rather be presented in contexts, since the context is what tells you it is a part-to-part ratio. Part-to-part ratios occur in geometry (corresponding parts of similar figures and the ratio of the diagonal of a square to its side), algebra (slope of a line is a ratio of its rise to its run), and statistics & probability (the odds of an event occurring/not occurring is a part-to-part ratio).

Ratios can also show part-to-whole comparisons. For example, the ratio of girls in a class (8) to the number of students in the class (15) can be written as $\frac{8}{15}$. This can also be thought of as eight-fifteenths of the class. Percentages and probabilities are examples of part-to-whole ratios.

Many students view ratios and fractions as the same thing. While they are similar, they should be taught as overlapping concepts with certain distinctions. In his book, *Teaching Student Centered Mathematics*, Van de Walle gives three situations which illustrate these distinctions:

1. The ratio of cats to dogs at a pet store is $\frac{3}{5}$.
This ratio is not a fraction because fractions are not part-to-part ratios.
2. The ratio of cats to pets at the pet store is $\frac{3}{8}$.
This can be adapted to say that $\frac{3}{8}$ of the pets are cats. Since this is a part-to-whole ratio, it is both a ratio and a fraction.
3. Mario walked three-eighths of a mile ($\frac{3}{8}$ mile).
This is a fraction of a length and is not a ratio because there is not a multiplicative comparison.

There are two ways students should be able to think about ratio: as multiplicative comparisons and as composed units.

Multiplicative comparisons can be viewed in two ways. Example: Cup A holds 8 oz. Cup B holds 10 oz. The ratio of the two cups is 8 to 10, but this doesn't tell us about the relationship between the measures. To compare the cups multiplicatively:

The small cup holds eight-tenths as much as the large cup (it holds four-fifths as much).

The large cup holds ten-eighths as much as the short cup (it holds five-fourths or $1\frac{1}{4}$ as much).

“Composed units” refers to thinking of the ratio as one unit. This is similar to the idea of thinking of a group of ten “ones” as a unit of “ten.” For example, if lemons are 4 for \$1.00, you can think of this as a unit, then think about other ratios that would also be true for this given:

8 for \$2.00
16 for \$4.00
2 for \$0.50
1 for \$0.25

Any number of lemons can be priced by using these composed units.

For this task, it is important for students to make sense of the context and the rate for which they are asked to find. The ratio they are asked for in each context may be different than they anticipate, given the information. For example, with the goats, students may initially be thinking about pizzas per goat rather than goats per pizza which is what is asked for in that problem. The idea that ratios can be viewed in two ways is an important concept for students and builds students’ flexibility in reasoning with ratios.

The goal of this task is for students to find unit rates in different situations involving unusual units. Most students are familiar with miles per hour, but students are unlikely to have encountered the idea of pumpkins per hippo or goats per pizza.

To open the lesson, show two cups (as in the [example](#) above) and ask students to compare them. Students will likely say that one is smaller than the other or one is larger than the other. When you ask for a more specific comparison, students will ask for more information about the cups. Tell them that one holds 8 oz., and the other holds 10 oz. Ask for more comparisons. Students may say that the smaller cup holds 2 oz. less than the larger cup (this is an additive comparison). See proportional reasoning [misconceptions](#) section.

Through questioning, guide students to compare the capacities of the cups (8:10 or 10:8). Then, again through questioning, guide students to state the capacity of one cup in relation to the other (how many times more will cup B hold than cup A? What fractional part of one cup is the other?)

The following questions may help guide this discussion:

- How could you use the capacities of the cups to compare them?
- You’ve told me that cup B is bigger and by how much. **But, can you tell me how many times bigger one cup is than the other?**
- How else could you compare the capacities of the two cups? (What fractional part of cup B is cup A? And vice versa?)

During the work session, students should work with a partner to determine the ratios of each of the situations presented in the task below. The expectation should be set for students to represent each situation using a diagram, table, double number line, etc., as well as an equation to show their mathematical thinking.

During the closing or summary of the lesson, the teacher should facilitate a discussion of the problems, allowing students to compare solutions as well as strategies that allowed students to arrive at those solutions. The most efficient strategies should be shared toward the end, and a discussion of which strategies were most efficient in solving the problems (and why students think they were the most efficient) should be one goal of the closing of this lesson. Students need to see various strategies to connect their own understanding of a concept to other knowledge. It is important for even the most gifted students to pay attention to all strategies presented. Often, gifted students can know an answer, but have no idea how they arrived at the solution. Listening to all strategies can often point them in the direction and help them understand their own thinking.

DIFFERENTIATION

Extension:

Students who need an extension for this task should compare the ratios another way. For example, in the goat and pizza context, students can find the ratio of pizzas per goat as well. Students can also research other, possibly unusual, contexts where ratios are used and write new problems.

Intervention:

Students who need support should be asked to represent the situation with materials, creating a physical model of the problem. The teacher can then scaffold students' understanding of ratios. These models should be shared with the class during the classroom discussion at the end of the lesson.

Real??? World Ratios (Spotlight Task)

Imagine that 12 goats got into a dumpster behind a pizza parlor and ate 3 pizzas. How many goats per pizza would that be?

$$\frac{12}{3} \text{ or } 4 \text{ goats per pizza}$$



Every day 20 banks are robbed for a total of about \$50,000. How much money per bank is that?

$$\frac{50,000}{20} \text{ or } 2,500 \text{ dollars per bank}$$



In the world's longest running experiment, scientists have tried to capture tar pitch dripping on camera. In the past 86 years, 9 drops have formed. How many years per drop is that?

$$\frac{86}{9} \text{ or } 9\frac{5}{9} \text{ years per drop}$$

Hippos sometimes get to eat pumpkins as a special treat. If 3 hippos eat 5 pumpkins, how many pumpkins per hippo is that?

$$\frac{5}{3} \text{ pumpkins per hippo}$$



Real??? World Ratios (Spotlight Task)

Imagine that 12 goats got into a dumpster behind a pizza parlor and ate 3 pizzas. How many goats per pizza would that be?



Every day 20 banks are robbed for a total of about \$50,000. How much money per bank is that?



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Hippos sometimes get to eat pumpkins as a special treat. If 3 hippos eat 5 pumpkins, how many pumpkins per hippo is that?



Ratios and Rates

[Back to Task Table](#)

In this task, students will express ratios in various ways and, given non-unit rates, will determine unit rates.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
- 2. Reason abstractly and quantitatively.** Students understand the relationship between two quantities in order to express them mathematically. They use ratio and rate notation as well as visual models and contexts to demonstrate reasoning.
- 6. Attend to precision.** Students use and interpret mathematical language to make sense of ratios and rates.
- 7. Look for and make use of structure.** The structure of a ratio is unique and can be used across a wide variety of problem-solving situations. For instance, students recognize patterns that exist in ratio tables, including both the additive and multiplicative properties. In addition, students use their knowledge of the structures of word problems to make sense of real-world problems.
- 8. Look for and express regularity in repeated reasoning.** Students utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts. Students can generalize the relationship between representations, understanding that all formats represent the same ratio or rate.

ESSENTIAL QUESTIONS

- What information do I get when I compare two numbers using a ratio?
- What kinds of problems can I solve by using ratios?
- What are rates?
- How are unit rates helpful in solving real-world problems?
- How are ratios and rates similar and different?

A ratio can be expressed three ways:

- Using the fraction bar as in $\frac{2}{3}$
- Using a colon symbol as in 2:3
- Using the word “to” as in 2 to 3.

Write each ratio using the other two ways:

1. The ratio of 3 inches to 20 feet.

$$\frac{3 \text{ inches}}{20 \text{ feet}}; 3 \text{ inches} : 20 \text{ feet}$$

2. The ratio of 26 students: 1 class

$$\frac{26 \text{ students}}{1 \text{ class}}, 26 \text{ students to 1 class}$$

3. The ratio of $\frac{2 \text{ boys}}{3 \text{ girls}}$

$$2 \text{ boys} : 3 \text{ girls}; 2 \text{ boys to 3 girls}$$

When the denominator of a rate is 1, we call the rate a **unit rate**. We usually use the key word “per” or the division symbol / to indicate a unit rate. For example: If a student earns \$7.65 per hour, it is the same as \$7.65/hour, and means \$7.65 for every hour of work.

Find the unit rate for the following:

4. 120 eggs from 20 chickens *6 eggs/chicken*
5. \$55 for 20 people *\$2.75/person*
6. 250 miles in 4 hours *62.5 miles/hour*
7. 60 feet in 4 minutes *15 feet/minute*
8. 48 books for 16 students *3 books/student*
9. 56 children from 14 families *4 children/family*

Unit rates can also be used to solve problems.

10. Which is the better deal: 8 ounces of shampoo for \$0.89 or 12 ounces for \$1.47?

$\$.89/8 \text{ oz} = \$.0742/\text{oz}$ (≈ 7 cents per ounce); $\$1.47/12 \text{ oz} = \$.1225/\text{oz}$ (≈ 12 cents per ounce) therefore 8 ounces for \$0.89 is a better deal

11. Which is the better deal: 3 cans of soda for \$1.27 or 5 cans of soda for \$1.79

$\$1.27/3 \text{ cans} \approx 42 \text{ cents/can}$; $\$1.79/5 \text{ cans} \approx 36 \text{ cents/can}$, therefore 5 cans of soda for \$1.79 is a better deal

12. Which is the better deal: 10 pounds of hamburger for \$4.99 or 5 pounds of hamburger for \$2.69

$\$4.99/10 \text{ pounds} \approx \$.50/\text{pound}$; $\$2.69/5 \text{ pounds} \approx \$.54/\text{pound}$, therefore 10 pounds of hamburger for \$4.99 is a better deal

13. Which is traveling faster: Traveling 300 miles in 5 hours or traveling 250 miles in 4 hours

$300 \text{ miles}/5 \text{ hours} = 60 \text{ miles/hour}$; $250 \text{ miles}/4 \text{ hours} = 62.5 \text{ miles/hour}$, therefore 250 miles in 4 hours is faster

14. Which is traveling faster: Traveling 75 miles in 1 hour or traveling 280 miles in 3.5 hours

75 miles/hour ; $280 \text{ miles}/3.5 \text{ hours} = 80 \text{ miles/hour}$, therefore 280 miles/3.5 hours is faster

15. Which is traveling faster: Traveling 150 yards in 40 seconds or traveling 406 feet in 35 seconds

$150 \text{ yards}/40 \text{ seconds} = 450 \text{ feet}/40 \text{ seconds} = 3.75 \text{ yards/second}$ or $11.25 \text{ feet/second}$; $406 \text{ feet}/35 \text{ seconds} = \underline{11.6 \text{ feet/second}}$, therefore 406 yards in 35 seconds is faster (Did they observe that different units have to be converted to the same unit?)

Name _____

Ratios and Rates

A ratio can be expressed three ways:

- Using the fraction bar as in $\frac{2}{3}$
- Using a colon symbol as in 2:3
- Using the word “to” as in 2 to 3.

Write each ratio using the other two ways:

1. The ratio of 3 inches to 20 feet.
2. The ratio of 26 students: 1 class
3. The ratio of $\frac{2 \text{ boys}}{3 \text{ girls}}$

When the denominator of a rate is 1, we call the rate a **unit rate**. We usually use the key word **per** or the division symbol (/) to indicate a unit rate. For example: If a student earns \$7.65 per hour, it is the same as \$7.65/hour, and means \$7.65 for every hour of work.

Find the unit rate for the following:

4. 120 eggs from 20 chickens
5. \$55 for 20 people
6. 250 miles in 4 hours

7. 60 feet in 4 minutes

8. 48 books for 16 students

9. 56 children from 14 families

Unit rates can also be used to solve problems.

10. Which is the better deal: 8 ounces of shampoo for \$0.89 or 12 ounces for \$1.47

11. Which is the better deal: 3 cans of soda for \$1.27 or 5 cans of soda for \$1.79

12. Which is the better deal: 10 pounds of hamburger for \$4.99 or 5 pounds of hamburger for \$2.69

13. Which is traveling faster: Traveling 300 miles in 5 hours or traveling 250 miles in 4 hours

14. Which is traveling faster: Traveling 75 miles in 1 hour or traveling 280 miles in 3.5 hours

15. Which is traveling faster: Traveling 150 yards in 40 seconds or traveling 406 feet in 35 seconds

Constant Dimensions

[Back to Task Table](#)

Adapted from NCTM Illuminations

In this inquiry-based task, students will discover a proportional relationship between the length and width of a rectangle and extrapolate that relationship to other units of measure.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
- 2. Reason abstractly and quantitatively.** Students understand the relationship between two quantities in order to express them mathematically. They use Ratio and rate notation as well as visual models and contexts to demonstrate reasoning.
- 3. Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding appropriateness of representations given ratio and rate contexts. For example, does a tape diagram adequately represent a given ratio scenario.
- 4. Model with mathematics.** Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.
- 5. Use appropriate tools strategically.** Students choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.
- 6. Attend to precision.** Students use and interpret mathematical language to make sense of ratios and rates.
- 7. Look for and make use of structure.** The structure of a ratio is unique and can be used across a wide variety of problem-solving situations. For instance, students recognize patterns that exist in ratio tables, including both the additive and multiplicative properties. In addition, students use their knowledge of the structures of word problems to make sense of real-world problems.
- 8. Look for and express regularity in repeated reasoning.** Students utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts. Students can generalize the relationship between representations, understanding that all formats represent the same ratio or rate.

INTRODUCTION

Students will measure the length and width of a rectangle using both standard and non-standard units of measure. In addition to providing measurement practice, this lesson allows students to discover that the ratio of length to width of a rectangle is constant, in spite of the units. For many middle school students, this discovery is surprising.

Students will:

- Critique various units of measure based on their appropriateness for this particular activity.
- Plot ordered pairs, analyze and make predictions.
- Draw conclusions about the relationship of two dimensions based on collected data.

MATERIALS

Rulers (both inches and centimeters)

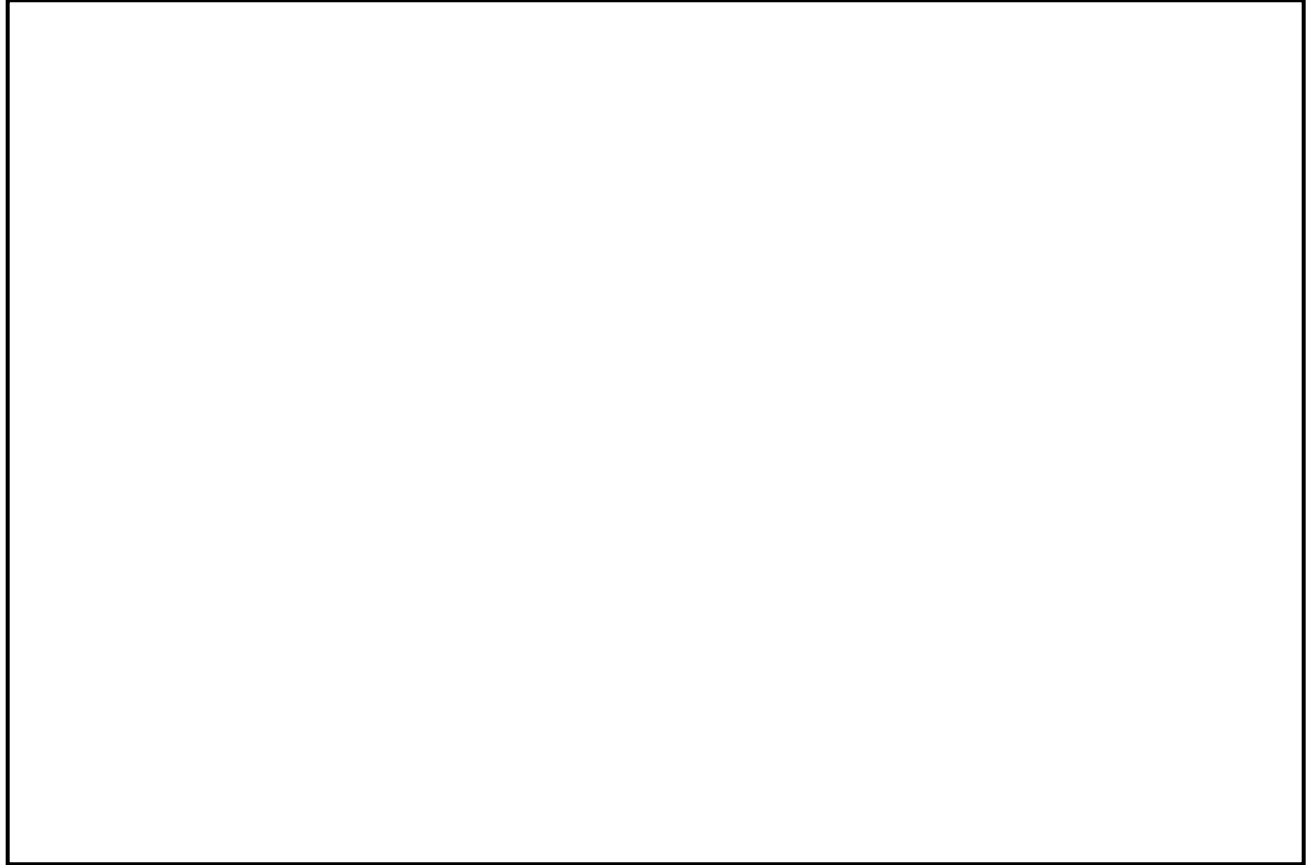
Alternate units of measure (pennies, paper clips, M&M's, beads, width of index finger, width of pencil)

Ask students to measure the length and width of the rectangle in both inches and centimeters. They should record their measurements in the chart below. At this point, it might be helpful to ask students to share their measurements to be sure students are on the right track.

After students have measured the rectangle in inches and centimeters, distribute alternative units of measure, such as paper clips, M&M's, pennies, beads, etc.

1. Measure the rectangle using five different units and record in the chart below.

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Unit	Length	Width
Inches	<i>6 $\frac{3}{4}$ inches</i>	<i>4 $\frac{1}{2}$ inches</i>
Centimeter	<i>17.2 cm</i>	<i>11.4 cm</i>
<i>Answers will vary</i>		

2. Create a list of ordered pairs to represent the measurements of the rectangle you found (L, W)

(6 $\frac{3}{4}$, 4 $\frac{1}{2}$)

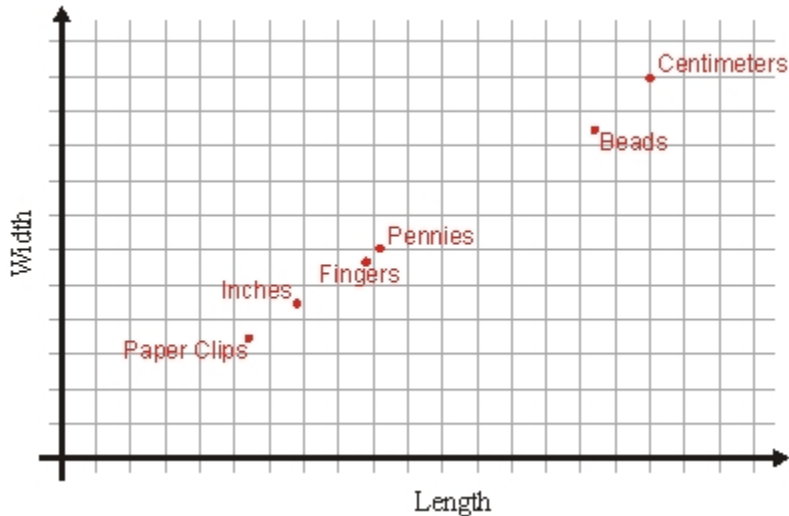
(17.2, 11.4)

Etc.

Georgia Department of Education
Georgia Standards of Excellence Framework
GSE Grade 6 Mathematics • Unit 2

3. Create a scatter plot of measurements. Use graph paper.

You may want to complete a scatterplot based on student measurements. If pairs of students have used different units of measure, you may be able to display a scatterplot with more than six points by aggregating the measurements from the entire class.



4. Do the points appear to be random, or do they seem to follow a pattern?

The points seem to occur in a straight line

5. What appears to be the ratio of Length to Width? $3/2$
6. If someone used gumballs to measure the length and width, and their ordered pair were placed at $(22, 10)$, would we suspect they made a good measurement? What if the ordered pair had the coordinates $(16, 10.5)$ would that be reasonable? What is your reasoning?

The point $(22, 10)$ seems to indicate an incorrect measurement because $22/10$ is not equivalent to $3/2$. The point $(16, 10.5)$ does seem reasonable because the ratio is close to $3/2$.

7. The length of the rectangle measured approximately 7.9 nickels. What is the width of the rectangle in nickels?

Approximately 5.3 nickels

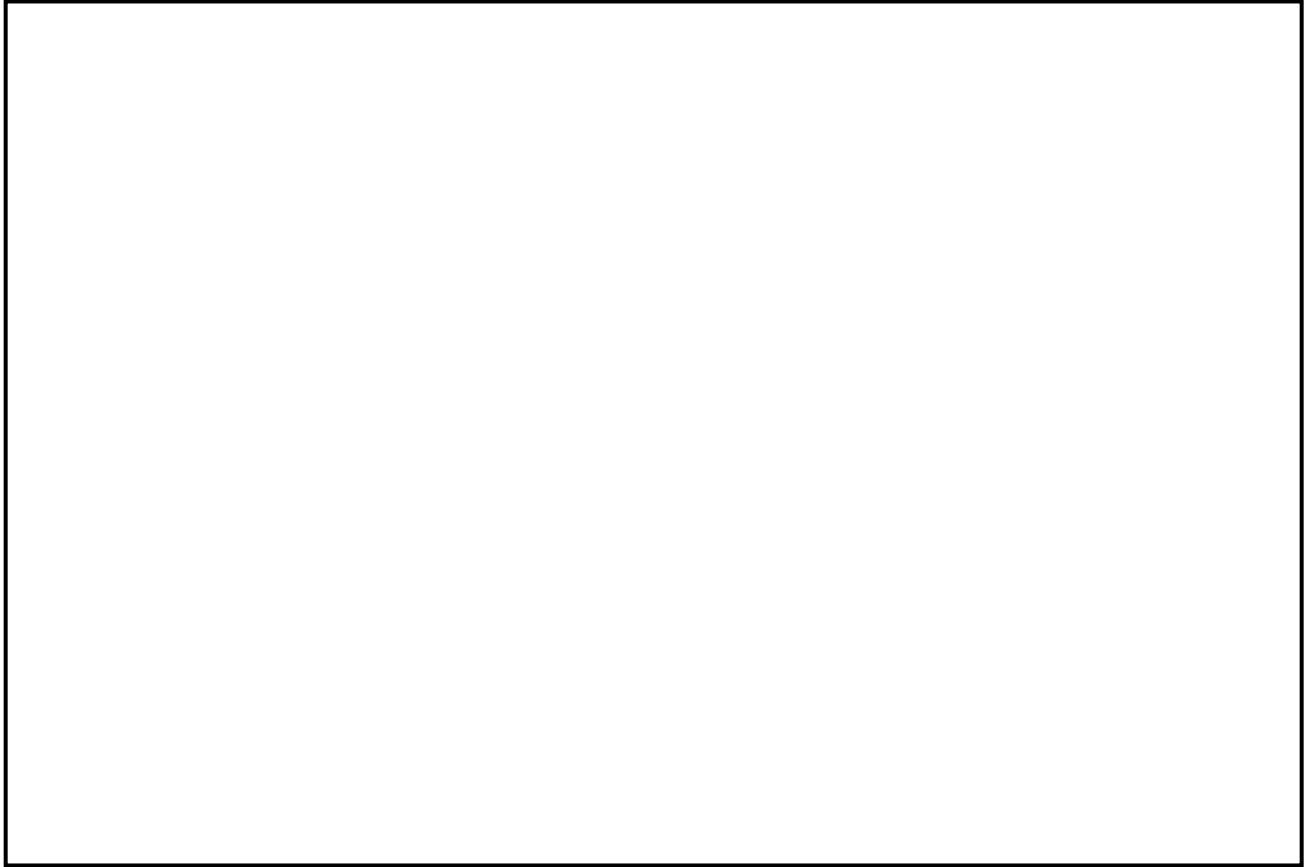
8. If the length of the rectangle is 15 wooches, determine the width of the rectangle in wooches.

Approximately 10 wooches

Name: _____

Constant Dimensions

1. Measure the rectangle using five different units and record in the chart below.



Unit	Length	Width
Inches		
Centimeter		

2. Create a list of ordered pairs to represent the measurements of the rectangle you found (L,W)

3. Create a scatter plot of measurements. Use graph paper.

4. Do the points appear to be random, or do they seem to follow a pattern? Explain.

5. What appears to be the ratio of Length to Width?

6. If someone used gumballs to measure the length and width, and their ordered pair were placed at (22, 10), would we suspect they made a good measurement? What if the ordered pair had the coordinates (16, 10.5) would that be reasonable? What is your reasoning?

7. The length of the rectangle measured approximately 7.9 nickels. What is the width of the rectangle in nickels?

8. If the length of the rectangle is 15 wooches, determine the width of the rectangle in wooches.

** How Many Noses Are In Your Arms?

[Back to Task Table](#)

Adapted from: http://www.pbs.org/teachers/mathline/lessonplans/msmp/noses/noses_procedure.shtm

In this task, students will apply the concept of ratio and proportion to determine the length of the Statue of Liberty's torch-bearing arm.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. *For example, given $1 \text{ in.} = 2.54 \text{ cm}$, how many centimeters are in 6 inches?*

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students use proportional reasoning to find the lengths on the Statue of Liberty.
- 2. Reason abstractly and quantitatively.** Students understand the relationship between the two different quantities; The Statue of Liberty and their body measurements. Students use this relationship to set up proportions.
- 3. Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding appropriateness of representations given ratio contexts.
- 4. Model with mathematics.** Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.
- 8. Look for and express regularity in repeated reasoning.** Students utilize repeated reasoning by applying their knowledge of proportions to solve the problem. .

TEACHER NOTES

Students view a photo of the Statue of Liberty and are asked how long the arm would be if the nose measures 4 feet 6 inches. Given chart paper, string, and rulers, students develop their own strategy for finding the solution. They measure the length of their nose and the length of their arm and form a ratio. Using proportions, students compute the length of the statue's arm. Group results are displayed and compared. The actual length of the Statue of Liberty's arm is located in the almanac and compared to the lengths determined by the students.

MATERIALS

Each Group:

Rulers, String, Almanac, Calculators, Chart paper and colored markers

1. Measure your arm span from finger tip to finger tip. Measure your height. Find the ratio of your arm span to your height.

Ratio is 1:1

2. Measure the length of your foot and the distance around your fist. Find the ratio of the length of your foot to the distance around your fist.

Ratio is 1:1

3. Using the picture of the Statue of Liberty and the fact that her nose measures 4 feet 6 inches from the bridge to the tip, determine the length of the Statue of Liberty's right arm, the one holding the torch.

The actual length of her arm is 42 feet

4. What strategy did you use to determine the length of the Statue of Liberty's right arm?

Answers will vary, share the variety of strategies with the class. Explore possible reasons for discrepancies in determining the length of the arm.

5. Is the ratio of the measurement of the length of your nose to the length of your arm the same as the ratio of the Statue of Liberty's?

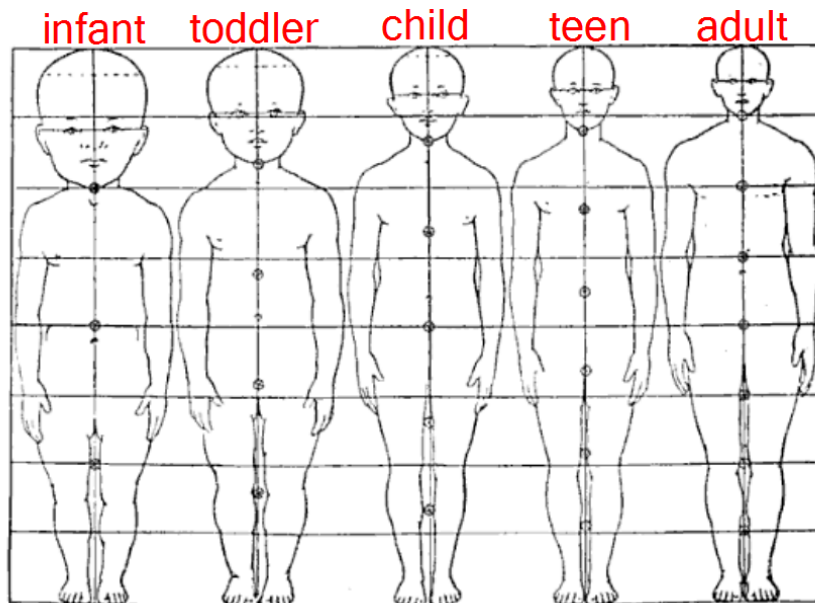
Answers will vary



<http://channeleve.co.uk/wp-content/uploads/2015/02/Statue-of-Liberty.jpg>

Extension: Explore a variety of body part ratios by measuring and then making conjectures about the “average” ratio. For instance, have students measure the length of their head, and then measure their height (using the same unit). Then have students conjecture about what might be the typical ratio for head length to height. The following illustration could be used as an activator for such an investigation:

Human Body Ratios



What do you notice about human body part ratios over time? Identify some ratios that stand out to you.

Name _____

How Many Noses Are In Your Arms?

1. Measure your arm span from fingertip to fingertip. Measure your height. Find the ratio of your arm span to your height.
2. Measure the length of your foot and the distance around your fist. Find the ratio of the length of your foot to the distance around your fist.
3. Using the picture of the Statue of Liberty and the fact that her nose measures 4 feet 6 inches from the bridge to the tip, determine the length of the Statue of Liberty's right arm, the one holding the torch.
4. What strategy did you use to determine the length of the Statue of Liberty's right arm?
5. Is the ratio of the measurement of the length of your nose to the length of your arm the same as the ratio of the Statue of Liberty's?

Reaching the Goal

[Back to Task Table](#)

In this task, students will compare percents with other forms of rational numbers and other types of numbers

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the relationship between fractions, decimals, and percent to solve the problems.
- 3. Construct viable arguments and critique the reasoning of others.** Students explain the relationship between fractions, decimals, and percent.
- 4. Model with mathematics.** Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.
- 6. Attend to precision.** Students use and interpret mathematical language to make sense of ratios and rates.

ESSENTIAL QUESTIONS

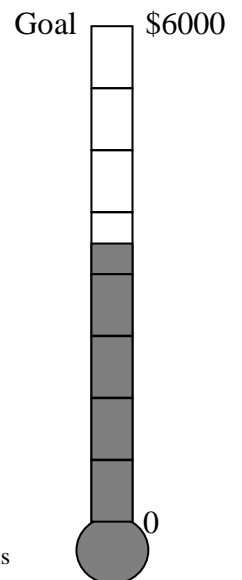
- How can I tell which form of a rational number is most appropriate in a given situation?
- What information do I get when I compare two numbers using a ratio?
- What kinds of problems can I solve by using ratios?
- What is the relationship between fractions, decimals, and percent?

MATERIALS

- colored pencils

TASK COMMENTS

In this task, students will compare percents with other forms of rational numbers and other types of numbers. Students will also compare the equivalence.



Reaching the Goal

After looking at the scale to the right...

Michael said, "We have reached $\frac{5}{8}$ of our goal."

Juan said, "I think we have earned about 60% of the \$6,000 we need."

Fiona said, "We have earned about \$3,500."

Nathan said, "You are all close, but none of you are correct."

Nathan is correct.

- a) Represent the amount earned as a fraction, as a decimal, as a percent, and as a dollar amount.

Solution

Students should notice that the diagram is divided into 8 equal parts. The shaded area covers approximately 4 and one-half eighths or $\frac{9}{16}$. The amount earned is $\frac{9}{16}$ as a fraction, .5625 as a decimal, and 56.25%. The dollar amount is $\frac{9}{16}$ of \$6000 or \$3375.

- b) Show how you know that the *fraction*, the *percent*, the *decimal*, and the *dollar amount* answers are all equivalent in this situation.

Students should show how they converted from one form of rational number to another. They should also show their work in obtaining the dollar amount.

- c) Which 3 of the amounts are *always* equivalent to each other? Why?

The fraction, decimal and percent are always equivalent because they represent the same part of 1.

- d) Which amount is *not always* equivalent to the others? Why not?

The dollar amount is not always equivalent to the others because the total dollar amount, which represents one whole, can change. For example, suppose the goal was \$10,000 instead of \$6000. Then the dollar amount would be different.

Name _____

Reaching the Goal

After looking at the scale to the right...

Michael said, "We have reached $\frac{5}{8}$ of our goal."

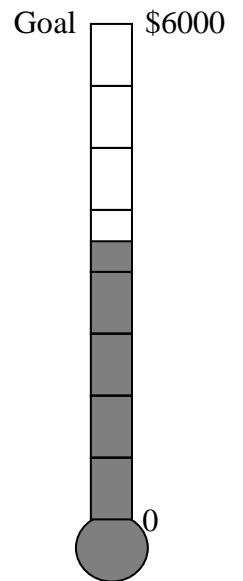
Juan said, "I think we have earned about 60% of the \$6,000 we need."

Fiona said, "We have earned about \$3,500."

Nathan said, "You are all close, but none of you are correct."

Nathan is correct.

- Represent the amount earned as a fraction, as a decimal, as a percent, and as a dollar amount.
- Show how you know that the *fraction*, the *percent*, the *decimal*, and the *dollar amount* answers are all equivalent in this situation.
- Which 3 of the amounts are *always* equivalent to each other? Why?
- Which amount is *not always* equivalent to the others? Why not?



***Free Throw Warm-up**

[Back to Task Table](#)

In this task, students will shoot “free throws” while analyzing and conjecturing about rate increases and decreases.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
- 2. Reason abstractly and quantitatively.** Students will analyze how constancy and change in the numerator and denominator of a fraction translate to rate increases or decreases.
- 3. Construct viable arguments and critique the reasoning of others.** Students explain the relationship between ratios and percentages.
- 8. Look for and express regularity in repeated reasoning.** Students observe how rates increase or decrease based on the relationship between free throws made and free throws attempted.

ESSENTIAL QUESTIONS

- What information do I get when I compare two numbers using a ratio?
- What kinds of problems can I solve by using ratios?

TASK COMMENTS

This task serves as a prelude to the subsequent task, Free Throws. The purpose of this task is to have students participate in an engaging, hands-on exploration of how rates change when “free throws” are made or missed. The concepts emphasized in this task will enable students to realize that rates always increase when a shot is made and rates always decrease when shots are missed. In other words, increasing the numerator and denominator by the same amount always results in a rate increase, while increasing the denominator while keeping the numerator the same always results in a rate decrease. Students will also investigate these rates in multiple forms (e.g., as fractions, decimals, and percents).

MATERIALS NEEDED

- Large bin (for shooting balls into; place a blanket or pillow in the bin to reduce bouncing)
- Ball (tennis ball, foam ball, etc.)
- Free Throw Warm-up student recording sheet (attached)
- Calculators

DIRECTIONS

Tell students that they will participate in a class free throw activity. Place a large bin in the center of the classroom. It will be helpful to place a blanket or pillow inside the bin to reduce the likelihood of the ball bouncing out. You may want to place the bin in a location that will not be too easy or too difficult to make all of the shot attempts. Ideally, place the bin in a location that will allow a theoretical probability of about 50% of making each shot (this will allow rates/percentages to ebb and flow more).

Distribute *Free Throw Warm-up* student recording sheet and calculators.

Allow each student to take 1 shot attempt (using a tennis ball, foam ball, or some other type of ball that is not likely to bounce out of the bin or cause damage in the classroom). After each student takes a shot, have students record the total number of shots made, and then express the rate of shots made to shots attempted as a fraction, decimal, and percent. Allow students to use a calculator to convert fractions to decimals/percents. You may want to tell students to round all decimals to the nearest thousandth and percents to the nearest tenth of a percent.

Be sure to ask students if they can predict whether the rate will increase or decrease **before** they even calculate the decimal/percent. With repeated trials, all students should realize that free throw rates/percents *always* increase when a shot is made and they *always* decrease when shots are missed.

It is very important to pause intermittently and pose questions such as:

- If we make the next 3 shot attempts what will be our shot percentage?
- If we make ___ of the next ___ shots, what will be our shot percentage?
- How many shots will we need to make in the next ___ attempts to increase our shot percentage to ___?
- What is the greatest number of shots we can miss in the next ___ attempts in order to avoid falling below ___ %?
- If we make ___ out of the next ___ shot attempts, will our shot percentage increase or decrease?
- What could be the greatest rate/percentage of shots made from this point out? How do you know?
- What could be the lowest rate/percentage of shots made from this point out? How do you know?

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FREE THROW WARM-UP

	Shots Made		Shots Attempted	Fraction	Decimal	Percent	Rate Increase or Decrease?
Trial 1		out of	1				
Trial 2		out of	2				
Trial 3		out of	3				
Trial 4		out of	4				
Trial 5		out of	5				
Trial 6		out of	6				
Trial 7		out of	7				
Trial 8		out of	8				
Trial 9		out of	9				
Trial 10		out of	10				
Trial 11		out of	11				
Trial 12		out of	12				
Trial 13		out of	13				
Trial 14		out of	14				
Trial 15		out of	15				
Trial 16		out of	16				
Trial 17		out of	17				
Trial 18		out of	18				
Trial 19		out of	19				
Trial 20		out of	20				
Trial 21		out of	21				
Trial 22		out of	22				
Trial 23		out of	23				
Trial 24		out of	24				
Trial 25		out of	25				
Trial 26		out of	26				
Trial 27		out of	27				
Trial 28		out of	28				
Trial 29		out of	29				
Trial 30		out of	30				

Free Throws

[Back to Task Table](#)

In this task, students will solve problems that feature comparison of various rates.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
- 3. Construct viable arguments and critique the reasoning of others.** Students explain the relationship between ratios and percentages.

ESSENTIAL QUESTIONS

- What information do I get when I compare two numbers using a ratio?
- What kinds of problems can I solve by using ratios?

TASK COMMENTS

In this task, students will use ratios to compare two quantities and solve a real-world problem.

1. Juan made 13 out of 20 free throws. If Bonita shoots 25 free throws, what's the minimum number she has to make in order to have a better free-throw percentage than Juan?
Bonita would need to have made 16 $\frac{1}{4}$ free throws to have the same free-throw percentage than Juan. She cannot make $\frac{1}{4}$ of a free throw so therefore she would need to make 17 free throws and would have a better free throw percentage.
2. Juan continues to shoot free throws. How many free throws would Juan need to make out of fifty to have the same percentage that Bonita now has?
Juan has shot twice as many free throws so he would need to make twice as many as Bonita or 34 free throws.
3. Can Bonita continue to have the same percentage if she shoots 60 free throws? If she shoots 75 free throws? If yes, how many free throws does she need to make?
If she shoots 60 free throws she would need to make 40 $\frac{4}{5}$ free throws. Since you cannot make a part of a free throw she would not be able to have the same percentage. If she shoots 75 free throws she would need to make three times as many as she currently has to maintain her free throw percentage or 51 made.

Name _____

Free Throws

1. Juan made 13 out of 20 free throws. If Bonita shoots 25 free throws, what's the minimum number she has to make in order to have a better free-throw percentage than Juan?
2. Juan continues to shoot free throws. How many free throws would Juan need to make out of fifty to have the same percentage that Bonita now has?
3. Can Bonita continue to have the same percentage if she shoots 60 free throws? If she shoots 75 free throws? If yes, how many free throws does she need to make?

* **Comparing Rates**

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Adapted from *Navigating Through Number and Operations in Grades 6-8* (2006): NCTM.

In this task, students will reason about changes in rates and make generalizations about what contributes to changes in rates and how.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
- 3. Construct viable arguments and critique the reasoning of others.** Students explain the relationship between given information and changes in rates.
- 6. Attend to precision.** Students must attend to different units being compared in ratios, being sure not to mix them up.
- 8. Look for and express regularity in repeated reasoning.** Students will make generalizations about rate changes based on patterns and relationships they notice in the task scenarios.

ESSENTIAL QUESTIONS

- What information do I get when I compare two numbers using a ratio?
- How can I predict whether a rate will increase, decrease, or stay the same?
- What information is necessary to determine if and how a rate might change?

TASK COMMENTS

In this task, students will reason about changes in rates using real-world scenarios. Have students work in groups. They should read the scenarios and then discuss how to classify them into 1 of 4 possible categories: 1) Rate Increases; 2) Rate Decreases; 3) Rate Stays the Same; and 4) Not Enough Information to Determine. It will be important for students to engage in strategies such as guess-and-check, substitution of “made up” numbers, rate tables, etc. In the end, during a whole group discussion, students should be asked to make generalizations about changes in rates (e.g., what changes *always* result in a rate decrease?).

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<p style="text-align: center;">Problem #1</p> <p>Damien noticed that the cost of a gallon of gas changed from last week. Yesterday, he paid more money for fewer gallons of gas. How did the cost per gallon change?</p> <p style="text-align: center;"><i>Rate Increase</i></p>	<p style="text-align: center;">Problem #2</p> <p>In last night’s game, Ray attempted more free throws than he did the night before, but he made the same amount of shots on both nights. Did Ray’s free throw percentage increase, decrease, or stay the same – or can you not tell?</p> <p style="text-align: center;"><i>Rate Decrease</i></p>
<p style="text-align: center;">Problem #3</p> <p>Michael jogs every day. He keeps a record of the distance and time it takes him to complete his runs. On Sunday, he ran a greater distance and a longer time than he did the day before. How did his speed change?</p> <p style="text-align: center;"><i>Not Enough Information to Determine</i></p>	<p style="text-align: center;">Problem #4</p> <p>Keira is making sweet tea. She mixes some sugar with tea. After tasting it, she decides to change the recipe by increasing the amount of sugar and decreasing the amount of water. Does the tea now taste sweeter, less sweet, or the same – or can you not tell?</p> <p style="text-align: center;"><i>Sweeter</i></p>
<p style="text-align: center;">Problem #5</p> <p>Today, Damien paid less money for the same amount of gas he purchased yesterday. How did the cost per gallon change?</p> <p style="text-align: center;"><i>Rate Decrease</i></p>	<p style="text-align: center;">Problem #6</p> <p>In tonight’s game, Ray took more free throw attempts and made more shots than the night before. Did Ray’s free throw percentage increase, decrease, or stay the same – or can you not tell?</p> <p style="text-align: center;"><i>Not Enough Information to Determine</i></p>
<p style="text-align: center;">Problem #7</p> <p>On Monday, Michael ran the same distance he ran on Sunday, but today’s run took longer. Did his speed increase, decrease, or stay the same – or can you not tell?</p> <p style="text-align: center;"><i>Rate Decrease</i></p>	<p style="text-align: center;">Problem #8</p> <p>Kiera decided to make more tea. This time, she used twice as much sugar as her recipe in problem 4 and she also doubled the amount of water. What happened to the taste of her new tea compared with the taste of the tea she made in problem 4? Does the tea taste sweeter, less sweet, or the same – or can you not tell?</p> <p style="text-align: center;"><i>Stays the Same</i></p>

Look back at each scenario and the answers you provided. Now, try to make a generalization about how rates increase, decrease, and stay the same. Use the phrases **Rate Increase**, **Rate Decrease**, **Rate Stays the Same**, or **Not Enough Information to Determine** to evaluate each statement below. Justify your answers by providing examples and/or counterexamples. You may wish to create your own rate tables to help determine how the rates compare for each question.

1. For the rate a/b , what happens when both a and b increase?
Not Enough Information to Determine
2. For the rate a/b , what happens when both a and b decrease?
Not Enough Information to Determine
3. For the rate a/b , what happens when a increases and b decreases?
Rate Increase
4. For the rate a/b , what happens when a decreases and b increases?
Rate Decrease
5. For the rate a/b , what happens when a increases and b stays the same?
Rate Increase
6. For the rate a/b , what happens when a decreases and b stays the same?
Rate Decrease
7. For the rate a/b , what happens when a stays the same and b increases?
Rate Decrease
8. For the rate a/b , what happens when a stays the same and b decreases?
Rate Increase

Comparing Rates

Name _____

Directions: Read each problem. With a partner or partners, discuss whether each rate changes, and if so, how. You must classify each problem into one of four categories: **Rate Increases, Rate Decreases, Rate Stays the Same, and Not Enough Information to Determine.** If you decide there is not enough information to determine how the rate changes, you must justify your answer with a possible example.

Problem #1 Damien noticed that the cost of a gallon of gas changed from last week. Yesterday, he paid more money for fewer gallons of gas. How did the cost per gallon change?	Problem #2 In last night’s game, Ray attempted more free throws than he did the night before, but he made the same amount of shots on both nights. Did Ray’s free throw percentage increase, decrease, or stay the same – or can you not tell?
Problem #3 Michael jogs every day. He keeps a record of the distance and time it takes him to complete his runs. On Sunday, he ran a greater distance and a longer time than he did the day before. How did his speed change?	Problem #4 Keira is making sweet tea. She mixes some sugar with tea. After tasting it, she decides to change the recipe by decreasing the amount of sugar and increasing the amount of water. Does the tea now taste sweeter, less sweet, or the same – or can you not tell?
Problem #5 Today, Damien paid less money for the same amount of gas he purchased yesterday. How did the cost per gallon change?	Problem #6 In tonight’s game, Ray took more free throw attempts and made more shots than the night before. Did Ray’s free throw percentage increase, decrease, or stay the same – or can you not tell?
Problem #7 On Monday, Michael ran the same distance he ran on Sunday, but today’s run took longer. Did his speed increase, decrease, or stay the same – or can you not tell?	Problem #8 Kiera did not like the tea she made so she changed her mixture once more. This time, she used the same amount of sugar as her recipe in problem 4 but decreased the amount of water. What happened to the taste of her new tea compared with the taste of the tea she made in problem 4? Did the tea taste sweeter, less sweet, or the same – or can you not tell?

Look back at each scenario and the answers you provided. Now, try to make a generalization about how rates increase, decrease, and stay the same. Use the phrases **Rate Increase**, **Rate Decrease**, **Rate Stays the Same**, or **Not Enough Information to Determine** to evaluate each statement below. Justify your answers by providing examples and/or counterexamples. You may wish to create your own rate tables to help determine how the rates compare for each question.

1. For the rate a/b , what happens when both a and b increase?

2. For the rate a/b , what happens when both a and b decrease?

3. For the rate a/b , what happens when a increases and b decreases?

4. For the rate a/b , what happens when a decreases and b increases?

5. For the rate a/b , what happens when a increases and b stays the same?

6. For the rate a/b , what happens when a decreases and b stays the same?

7. For the rate a/b , what happens when a stays the same and b increases?

8. For the rate a/b , what happens when a stays the same and b decreases?

Sharing Costs – Traveling To School-Formative Assessment Lesson

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Source: *Formative Assessment Lesson Materials from Mathematics Assessment Project*
<http://map.mathshell.org/materials/download.php?fileid=1366>

In this problem-based task, students will reason about rates and ratios to determine and justify a solution to an open-ended problem.

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:

<http://www.map.mathshell.org/materials/background.php?subpage=formative>

The task, *Sharing Costs: Traveling to School*, is a Formative Assessment Lesson (FAL) that can be found at the website: <http://map.mathshell.org/materials/lessons.php?taskid=489&subpage=problem>

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:

<http://map.mathshell.org/materials/download.php?fileid=1366>

STANDARDS FOR MATHEMATICAL CONTENT

Understand ratio concepts and use ratio reasoning to solve problems.

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

MGSE6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. *For example, given 1 in. = 2.54 cm, how many centimeters are in 6 inches?*

STANDARDS FOR MATHEMATICAL PRACTICE

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.

Optimizing Security Cameras - Formative Assessment Lesson

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Source: *Formative Assessment Lesson Materials from Mathematics Assessment Project*

<http://map.mathshell.org/materials/download.php?fileid=1354>

In this problem-based task, students will reason about ratios and percents to determine and justify a solution to an open-ended problem.

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:

<http://www.map.mathshell.org/materials/background.php?subpage=formative>

The task, *Optimizing: Security Cameras*, is a Formative Assessment Lesson (FAL) that can be found at the

website: <http://map.mathshell.org/materials/lessons.php?taskid=482&subpage=problem>

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:

<http://map.mathshell.org/materials/download.php?fileid=1354>

STANDARDS FOR MATHEMATICAL CONTENT

Understand ratio concepts and use ratio reasoning to solve problems.

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

MGSE6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. *For example, given 1 in. = 2.54 cm, how many centimeters are in 6 inches?*

STANDARDS FOR MATHEMATICAL PRACTICE

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Ice Cream or Cake?

[Back to Task Table](#)

In this problem-based task, students will reason about rates and percents.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
- 3. Construct viable arguments and critique the reasoning of others.** Students explain the relationship between ratios and percentages.

ESSENTIAL QUESTIONS

- What information do I get when I compare two numbers using a ratio?
- What kinds of problems can I solve by using ratios?
- Where do we see fractions, decimals, and percents being used in the real-world?

Suppose you survey all the students at your school to find out whether they like ice cream or cake better as a dessert, and you record your results in the contingency table below.

	ice cream	cake	totals
boys	82	63	145
girls	85	70	155
totals	167	133	300

- a) What percentage of students at your school prefers ice cream over cake?

We know that there are 300 students at the school and $167/300 \approx 56/100$ of them prefer ice cream. Therefore this means that about 56% of the students prefer ice cream.

- b) At your school, are those preferring ice cream more likely to be boys or girls?

Those who prefer ice cream are more likely to be girls since, in this survey, there are more girls who like ice cream than boys.

c) At your school, are girls more likely to choose ice cream over cake than boys are?

To identify the group more likely to choose ice cream over cake, we first find the percentage of girls who prefer ice cream and the percentage of boys who prefer ice cream.

There are 155 total girls, and 85 of them prefer ice cream. Calculate the percentage of the boys that prefer ice cream.

Therefore, boys (57%) are more likely than girls (55%) to prefer ice cream.

Even though an ice cream lover is more likely to be a girl than a boy, we cannot conclude that the chance of preferring ice cream is greater among girls than among boys. (This sort of false conclusion is common in public discourse.)

Name _____

Ice Cream or Cake?

Suppose you survey all the students at your school to find out whether they like ice cream or cake better as a dessert, and you record your results in the contingency table below.

	ice cream	cake	totals
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- a) What percentage of students at your school prefers ice cream over cake?

- b) At your school, are those preferring ice cream more likely to be boys or girls?

- c) At your school, are girls more likely to choose ice cream over cake than boys are?

Culminating Task: The Rocky Mountain Vacation Trip Problem

[Back to Task Table](#)

STANDARDS FOR MATHEMATICAL CONTENT

Understand ratio concepts and use ratio reasoning to solve problems.

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

MGSE6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. For example, given $1 \text{ in.} = 2.54 \text{ cm}$, how many centimeters are in 6 inches?

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
- 2. Reason abstractly and quantitatively.** Students understand the relationship between two quantities in order to express them mathematically. They use ratio and rate notation as well as visual models and contexts to demonstrate reasoning.
- 3. Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding appropriateness of representations given ratio and rate contexts. For example, does a tape diagram adequately represent a given ratio scenario?

4. Model with mathematics. Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.

5. Use appropriate tools strategically. Students choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.

6. Attend to precision. Students use and interpret mathematical language to make sense of ratios and rates.

7. Look for and make use of structure. The structure of a ratio is unique and can be used across a wide variety of problem-solving situations. For instance, students recognize patterns that exist in ratio tables, including both the additive and multiplicative properties. In addition, students use their knowledge of the structures of word problems to make sense of real-world problems.

8. Look for and express regularity in repeated reasoning. Students utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts. Students can generalize the relationship between representations, understanding that all formats represent the same ratio or rate.

ESSENTIAL QUESTIONS

- What kinds of problems can I solve by using ratios?
- How can I tell if a relationship is multiplicative?
- What is the difference between a multiplicative and an additive relationship?
- What are equivalent ratios?
- What are rates?
- How are unit rates helpful in solving real-world problems?
- How are ratios and rates similar and different?
- What are percentages?
- What information do I get when I compare two numbers using a ratio?

The Rocky Mountain Vacation Trip Problem

William was driving with his father, Mr. Pitts, into the Rocky Mountains. The distance to the camp site is 180 miles one way. Their car has a trip computer that gives information during the trip. William read the car’s manual to help him interpret what the different gauges were showing.

Note: It is important that students thoroughly understand the meaning of the dashboard display.

Understanding the Features of Your Trip Computer	
Outside Temp: The temperature outside the car (in degrees Fahrenheit)	Height: Altitude or height above sea level-in feet
Average Fuel Economy: The number of miles driven divided by the number of gallons used – miles per gallon (mpg) Trip start 00.0.	Distance to Empty: An estimate of the amount of miles you can drive with the fuel in the tank, taking into account the present rate of fuel consumption-in miles
Fuel: Fuel in tank (in gallons)	Trip Odometer: Distance driven on this trip (in miles)

Note: Some students will need reminding of ratio tables, proportional relationships, and constants of proportionality. Other students will need support in discussion of the quantities associated with car travel.

Start of the Trip:

TRIP COMPUTER		
Outside Temp 60°	Height 1,000 feet	Fuel 17 GAL
AVG. Fuel Economy 00.0 MPG	Distance to Empty 425 miles	Trip Odometer 000

After 20 minutes:

TRIP COMPUTER		
Outside Temp 58°	Height 1,275 feet	Fuel 16 GAL
Avg. Fuel Economy 15.0	Distance to Empty 240 miles	Trip Odometer 015 miles

1. What changed during the first 20 minutes? Explain the changes.
The outside temperature dropped 2 degrees, the altitude went up 275 feet, one gallon of gas was used at a rate of 15 mile/gallon. The distance to empty dropped 185 miles in 15 miles.

2. Explain how the distance to empty has dropped from 425 miles to 240 miles in just 20 minutes of driving time.
Instead of making the anticipated miles per gallon the car was only making 15 miles/gallon resulting in a drop of the number of miles the car could go on the remaining amount of gas.

3. Predict how the display may look after another 20 minutes.
Answers will vary based upon whether the students believe the car will continue to travel uphill as it did in the first 20 minutes. If so the air temp would drop to 56°, the height would be 1,550 feet, fuel would show 15 GAL, the Avg. Fuel Economy would not change, the Distance to Empty would be 225 and the Trip Odometer would be 30 miles.

Temperature

From the computer display, you can see that William and his father started their trip at 1,000 feet above sea level, with the outside temperature of 60°F. William’s father used a simple “rule of thumb” to get a feeling for the temperature up on the mountain.

“For every 3,000 feet you gain in altitude, the temperature drops 20°F.”

4. Twenty minutes into the trip, did the display agree with Mr. Pitts’ “rule of thumb”?
Using the “rule of thumb” we would expect the temperature would drop 2 °F for an increase of 300 ft. It is close.

5. William and his dad plan to camp at 8,000 feet. What should they expect the temperature to be at that altitude? Illustrate the relationship.
13⅓° F.

6. Complete the table.

Height (ft)	0	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000
Air Temp(°F)	<i>66⅔</i>	<i>60</i>	<i>53⅓</i>	<i>46⅔</i>	<i>40</i>	<i>33⅓</i>	<i>26⅔</i>	<i>20</i>	<i>13⅓</i>	<i>6⅔</i>	<i>0</i>

7. Is the relationship a proportional relationship? Explain
Yes, the ratio between each is the same.

8. What should the temperature be when they are at 6,500 feet?
The temperature should be half way between the temperature for 6,000 ft and 7,000 ft, or $23\frac{1}{3}^{\circ} F$
9. At what altitude will the temperature be at freezing ($32^{\circ}F$)?
According to the table, just slightly over 5,000 feet.

Fuel Consumption

From the car’s manual, William knows that at 55 mph on a level road, the car gets 25mpg. The car’s gas tank holds 17 gallons. Furthermore, William knows that the one-way distance to the camp site is 180 miles, and that there are no gas stations on the trip.

10. William says, “Dad, we have plenty of fuel to make the return trip.”
 Do you agree? Can you explain his reasoning?
To travel 180 miles at 25 mpg it should only take $7\frac{1}{5}$ gallons of gas.
11. When driving on the mountain road, the average fuel economy (mpg) goes down dramatically. Dad says, “When going uphill in the mountains, the car will get about 15 miles per gallon.”
- Explain why the display labeled “Distance to Empty” indicated 75 miles when William and his dad reached the camp site, 180 miles after starting.
To travel 180 miles going uphill at 15 mpg it would take 12 gallons of gas leaving 5 gallons in the tank. At 15 mpg the remaining 5 gallons would get you 75 miles.
12. Explain why it was still possible to make the return trip with so little fuel left.
The car would travel 125 miles on the remaining 5 gallons if they were traveling on level ground. Since they will be going downhill the car should get better gas mileage.
13. Copy and complete this table showing the fuel used on the uphill trip.

Journey (distance in miles)	0	15	30	60	120	135	150	180
Fuel Used (gallons)	0	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>9</i>	<i>10</i>	<i>12</i>
Fuel in Tank (gallons)	17	<i>16</i>	<i>15</i>	<i>13</i>	<i>9</i>	<i>8</i>	<i>7</i>	<i>5</i>

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14. Make a similar table showing the fuel used on the downhill trip, assuming the car got 45 mpg.

<i>Journey (distance in miles)</i>	<i>0</i>	<i>45</i>	<i>90</i>	<i>135</i>	<i>180</i>
<i>Fuel Used (gallons)</i>	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>
<i>Fuel in Tank (gallons)</i>	<i>5</i>	<i>4</i>	<i>3</i>	<i>2</i>	<i>1</i>

15. Will the car complete the round trip on one tank of gasoline, running at 15 mpg uphill and 45 mpg downhill? Justify your answer.

Yes, the car will complete the trip with a gallon to spare based upon the table above.

Name _____

The Rocky Mountain Vacation Trip Problem

William was driving with his father, Mr. Pitts, into the Rocky Mountains. The distance to the camp site is 180 miles one way. Their car has a trip computer that gives information during the trip. William read the car’s manual to help him interpret what the different gauges were showing.

Understanding the Features of Your Trip Computer	
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Fuel: Fuel in tank (in gallons)	Trip Odometer: Distance driven on this trip (in miles)

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After 20 minutes:

TRIP COMPUTER		
Outside Temp 58°	Height 1,275 feet	Fuel 16 GAL
Avg. Fuel Economy 15.0	Distance to Empty 240 miles	Trip Odometer 015 miles

1. What changed during the first 20 minutes? Explain the changes.
2. Explain how the distance to empty has dropped from 425 miles to 240 miles in just 20 minutes of driving time.
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Temperature

From the computer display, you can see that William and his father started their trip at 1,000 feet above sea level, with the outside temperature of 60°F. William’s father used a simple “rule of thumb” to get a feeling for the temperature up on the mountain.

“For every 3,000 feet you gain in altitude, the temperature drops 20°F.”

4. Twenty minutes into the trip, did the display agree with Mr. Pitts’ “rule of thumb”?
5. William and his dad plan to camp at 8,000 feet. What should they expect the temperature to be at that altitude? Illustrate the relationship.
6. Complete the table.

Height(ft)	0	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000
Air Temp(°F)											

7. Is the relationship a proportional relationship? Explain.
8. What should the temperature be when they are at 6500 feet?
9. At what altitude will the temperature be at freezing (32°F)?

Fuel Consumption

From the car’s manual, William knows that at 55 mph on a level road, the car gets 25mpg. The car’s gas tank holds 17 gallons. Furthermore, William knows that the one-way distance to the camp site is 180 miles, and that there are no gas stations on the trip.

10. William says, “Dad, we have plenty of fuel to make the return trip.”
Do you agree? Can you explain his reasoning?
11. When driving on the mountain road, the average fuel economy (mpg) goes down dramatically. Dad says, “When going uphill in the mountains, the car will get about 15 miles per gallon.”
Explain why the display labeled “Distance to Empty” indicated 75 miles when William and his dad reached the camp site, 180 miles after starting.
12. Explain why it was still possible to make the return trip with so little fuel left.

13. Copy and complete this table showing the fuel used on the uphill trip.

Journey (distance in miles)	0	15	30	60	120	135	150	180
Fuel Used (gallons)	0							
Fuel in Tank (gallons)	17							

14. Make a similar table showing the fuel used on the downhill trip, assuming the car got 45 mpg.
15. Will the car complete the round trip on one tank of gasoline, running at 15 mpg uphill and 45 mpg downhill? Justify your answer.

***Unit Web Links**

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

<https://www.illustrativemathematics.org/content-standards/6/RP/A/1/tasks>

<http://nzmaths.co.nz/resource/ratios-and-rates>

<http://www.bbc.co.uk/schools/mathsfive/shockwave/games/fish.html>

<http://mathsnacks.com/baddate-en.html>

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

<https://www.illustrativemathematics.org/content-standards/6/RP/A/2/tasks>

<http://illuminations.nctm.org/Lesson.aspx?id=1110>

<http://nzmaths.co.nz/resource/ratios-and-rates>

<http://mathsnacks.com/baddate-en.html>

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

<http://www.learner.org/courses/learningmath/number/support/lmg8.pdf>

<https://www.illustrativemathematics.org/content-standards/6/RP/A/3/tasks>

<http://illuminations.nctm.org/Lesson.aspx?id=1658>

<http://illuminations.nctm.org/Lesson.aspx?id=2534>

[Understanding Tape Diagrams and Double Number Lines](#) (Teacher Link)

[Understanding Tape Diagrams](#) (Teacher Link)

<http://mathsnacks.com/baddate-en.html>

Dan Meyer:

<http://www.101qs.com/2841-nanas-paint-mixup>

<http://threeacts.mrmeyer.com/sugarpackets/>

<http://threeacts.mrmeyer.com/leakyfaucet/>

<http://threeacts.mrmeyer.com/nana/>

<http://mrmeyer.com/threeacts/superbear/>

<http://mrmeyer.com/threeacts/showervbath/>

<http://mrmeyer.com/threeacts/printjob/>

<http://threeacts.mrmeyer.com/splittime/>

Estimation180:

<http://www.estimated180.com/day-127.html>

<http://www.estimated180.com/day-128.html>

<http://www.estimated180.com/day-129.html>

<http://www.estimated180.com/day-130.html>

<http://www.estimated180.com/day-131.html>

<http://www.estimated180.com/day-132.html>

<http://www.estimated180.com/day-133.html>
<http://www.estimated180.com/day-134.html>

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

PARCC Prototype Task: [Gasoline Consumption](#)

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

NCTM Illuminations:

<http://illuminations.nctm.org/Lesson.aspx?id=1049>
<http://illuminations.nctm.org/Lesson.aspx?id=3170>
<http://illuminations.nctm.org/Lesson.aspx?id=960>

NZmaths:

<http://nzmaths.co.nz/search/node/percents>

OpenMiddle:

<http://www.openmiddle.com/interpreting-percentages/>

Estimation180:

<http://www.estimated180.com/day-129.html>
<http://www.estimated180.com/day-130.html>

MGSE6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. *For example, given 1 in. = 2.54 cm, how many centimeters are in 6 inches?*

OpenMiddle:

<http://www.openmiddle.com/convertng-between-fahrenheit-and-celsius/>

PARCC Prototype Item: [Slide Ruler](#)

[How Many Football Fields is 10 Miles?](#) by Andrew Stadel