## Mathematics

GSE Grade 6

## Unit 7: Rational Explorations: Numbers and their Opposites

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# Unit 7 <br> Rational Explorations: Numbers and their Opposites 

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## OVERVIEW

In this unit students will:

- understand that positive and negative numbers are used together to describe quantities having opposite directions or values.
- understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line.
- recognize that the opposite of the opposite of a number is the number itself.
- understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.
- recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- find and position integers and other rational numbers on a horizontal or vertical number line diagram.
- find and position pairs of integers and other rational numbers on a coordinate plane.
- understand ordering and absolute value of rational numbers.
- interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
- write, interpret, and explain statements of order for rational numbers in real-world contexts.
- understand the absolute value of a rational number as its distance from 0 on the number line
- interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
- distinguish comparisons of absolute value from statements about order.
- solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.


## STANDARDS ADDRESSED IN THIS UNIT

## STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

1. Make sense of problems and persevere in solving them. Students make sense of problems involving points and polygons in the coordinate plane.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about rational numbers with their visual representations. Students consider the values of these numbers in relation to distance (number lines).
3. Construct viable arguments and critique the reasoning of others. Students construct and critiques arguments regarding number line representations and the use of inequalities to represent real-world contexts.
4. Model with mathematics. Students use number lines to compare numbers and represent inequalities in mathematical and real-world contexts.
5. Use appropriate tools strategically. Students select and use tools such as two-color counters, number line models and the coordinate plane to represent situations involving positive and negative numbers.
6. Attend to precision. Students attend to the language of real-world situations to determine if positive or negative quantities/distances are being represented.
7. Look for and make use of structure. Students relate the structure of number lines to values of rational numbers as they use the coordinate plane.
8. Look for and express regularity in repeated reasoning. Students relate new experiences to experiences with similar contexts when studying positive and negative representations of distance and quantity. In the study of absolute value, students demonstrate repeated reasoning by showing that both positive and negative quantities represent the same distance from zero.

## STANDARDS FOR MATHEMATICAL CONTENT

## Apply and extend previous understandings of numbers to the system of rational numbers.

MGSE6.NS. 5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

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MGSE6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3)=3$, and that 0 is its own opposite.

MGSE6.NS.6b Understand signs of number in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

MGSE6.NS. 7 Understand ordering and absolute value of rational numbers.
MGSE6NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

MGSE6.NS.7b Write, interpret, and explain statements of order for rational numbers in realworld contexts.

MGSE6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a realworld situation.

MGSE6.NS.7d Distinguish comparisons of absolute value from statements about order.
MGSE6.NS. 8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

## Solve real-world and mathematical problems involving area, surface area, and volume.

MGSE6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply those techniques in the context of solving real-world mathematical problems.

## BIG IDEAS

- Negative numbers are used to represent quantities that are less than zero such as temperatures, elevation, scores in games or sports, and loss of income in business.
- Absolute value is useful in ordering and graphing positive and negative numbers.
- Positive and negative numbers are often used to solve problems in everyday life.
- Rational numbers are points on a number line.
- Numbers in ordered pairs indicate locations in quadrants of the coordinate plane.

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## ESSENTIAL QUESTIONS

- When are negative numbers used and why are they important?
- Why is it useful for me to know the absolute value of a number?
- When is graphing on the coordinate plane helpful?
- How do I use positive and negative numbers in everyday life?
- Where do I place positive and negative rational numbers on the number line?
- How do I use positive and negative numbers to represent quantities in real-world contexts?
- What are opposites, and how are opposites shown on a number line?
- How do statements of inequality help me place numbers on a number line?
- How can I use coordinates to find the distances between points?
- How can I use number lines to find the distances between points?
- How can I use absolute value to find the lengths of the sides of polygons on the coordinate plane?


## CONCEPTS/SKILLS TO MAINTAIN

In order for students to be successful, the following skills and concepts need to be maintained

- Changing between fractions and decimals
- Finding area of squares, rectangles, and triangles, and finding the perimeter of squares and rectangles.


## *FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

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Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

## Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

## STRATEGIES FOR TEACHING AND LEARNING

The purpose of this unit is to begin the study of the existence of negative numbers, their relationship to positive numbers, and the meaning and uses of absolute value. Starting with examples of having/owing and above/below zero sets the state of understanding that there is a mathematical way to describe opposites. Students should already be familiar with the counting numbers (positive whole numbers and zero), as well as with fractions and decimals (also positive). The students are now ready to understand that all numbers have an opposite. These special numbers can be shown on vertical or horizontal number lines, which then can be used to solve simple word problems. Demonstration of understanding of positives and negatives involves translating among words, numbers and models: give the words " 7 degrees below zero," showing it on a thermometer and writing -7; give -4 on a number line, writing a real-life example and mathematically as -4 . Number lines also give the opportunity to model absolute value as the distance from zero. Simple comparisons can be made and order determined. Or they can also be established and written mathematically: $-3^{\circ} \mathrm{C}>-5^{\circ} \mathrm{C}$ OR $-5^{\circ} \mathrm{C}<-3^{\circ} \mathrm{C}$. Finally, absolute values should be used to relate contextual problems to their meanings and solutions.

Using number lines to model negative numbers, prove the distance between opposites, and understand the meaning of absolute value easily transfers to the creation and usage of fourquadrant coordinate grids. Points can now be plotted in all four quadrants of a coordinate grid. Distances between numbers can be found by counting the distance between numbers on the grid. Computation (operations) with negative and positives is addressed in grade 7.

## INSTRUCTIONAL RESOURCES/TOOLS

- Vertical and Horizontal Number Lines
- Coordinate Graph Paper
- Colored Pencils
- http://www.shodor.org/interactivate/activities/GeneralCoordinates/
- https://www.mathsisfun.com/data/cartesian-coordinates-interactive.html


## SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. Note - Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.
http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary
http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.
http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

- Absolute value: The distance between a number and zero on the number line. The symbol for absolute value is shown in the equation $|-8|=8$.
- Cartesian Coordinate Plane: A plane containing two perpendicular axes ( $x$ and $y$ ) intersecting at a point called origin $(0,0)$.
- Coordinates: An ordered pair, $(x, y)$, that locates a point in a plane.
- Distance: amount of separation between 2 points.
- Inequality: Any mathematical sentence that contains the symbols > (greater than), < (less than), $\leq$ (less than or equal to), or $\geq$ (greater than or equal to).
- Integers: The set of whole numbers and their opposites $\{\ldots-3,-2,-1,0,1,2,3, \ldots\}$
- Magnitude: Greatness in size or amount
- Negative numbers: The set of numbers with a value less than zero
- Opposite number: Two different numbers that have the same absolute value. Example: 4 and -4 are opposite numbers because both have an absolute value of 4 . They are the same distance from zero, in opposite directions.
- Ordered Pair: A pair of numbers, $(x, y)$, that indicate the position of a point on the Cartesian coordinate Plane.
- Origin: The point of intersection of the vertical and horizontal axes of a Cartesian coordinate plane. The coordinates of the origin are ( 0,0 ).
- Polygon: A closed figure formed by three or more line segments.
- Positive number: The set of numbers whose value is greater than zero.
- Quadrant: One of the four regions on a Coordinate plane formed by the intersection of the x -axis and the y -axis.
- Rational number: The set of numbers that can be written in the form $\frac{a}{b}$ where $a$ and $b$ are integers and $b \neq 0$.
- Sign: a symbol that indicates whether a number is positive or negative. Example: in -4 , the ( - ) sign hows this number is read "negative four".
- $\mathbf{x}$-axis: The horizontal number line on the Cartesian coordinate plane.
- x-coordinate: The first number of in ordered pair; the position of a point relative to the vertical axis
- y-axis: The vertical number line on the Cartesian coordinate plane
- y-coordinate: The second number in an ordered pair; the position of a point relative to the horizontal axis


## FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

## SPOTLIGHT TASKS

For middle and high schools, each Georgia Standards of Excellence mathematics unit includes at least one Spotlight Task. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and researchbased pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky

## 3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

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| Task | Task Type Grouping Strategy | Content Addressed | Standards Addressed |
| :---: | :---: | :---: | :---: |
| $\frac{* * \text { What's Your }}{\text { Sign? }}$ | Learning / <br> Scaffolding Task Individual/Partner | Plotting numbers on a number line; Ordering integers; Negative and positive numbers; Absolute Value; Distance between two points on a number line | MGSE6.NS. 5 MGSE6.NS. 6 MGSE6.NS.6a MGSE6.NS.6c MGSE6.NS. 7 |
| Representing Rational Numbers on the Number Line | Learning / <br> Scaffolding Task Individual/Partner | Ordering rational numbers; Plotting numbers on a number line; Negative and positive numbers; Absolute Value; Opposites | MGSE6.NS. 5 <br> MGSE6.NS. 6 <br> MGSE6.NS.6a <br> MGSE6.NS.6c <br> MGSE6.NS. 7 |
| Symbols of Inequalities and the Coordinate System | Learning / <br> Scaffolding Task Individual/Partner | Plotting rational numbers on the number line; Negative and positive numbers; Plotting and naming points on a coordinate plane; Absolute Value; Distance between points | MGSE6.NS. 5 MGSE6.NS. 6 MGSE6.NS.6c MGSE6.NS. 7 MGSE6.NS.7a MGSE6.NS.7b MGSE6.NS. 8 |
| Graphing on the Coordinate Plane | Learning / <br> Scaffolding Task Individual/Partner | Plotting rational numbers on the number line; Negative and positive numbers; Plotting and naming points on a coordinate plane; Absolute Value; Distance between points; reflecting across the $x$ and $y$ axis; area and perimeter | MGSE6.NS. 5 <br> MGSE6.NS. 6 <br> MGSE6.NS.6a <br> MGSE6.NS.6b <br> MGSE6.NS.6c <br> MGSE6.NS. 7 <br> MGSE6.NS.7c <br> MGSE6.NS. 8 <br> MGSE6.G. 3 |
| Integers on the Coordinate Plane (FAL) | Formative <br> Assessment Lesson Individual/Partner | Plotting rational numbers on the number line; Negative and positive numbers; Plotting and naming points on a coordinate plane; | MGSE6.NS.6b MGSE6.NS.6c |
| Absolute Value and Ordering | Learning / <br> Scaffolding Task Individual/Partner | Absolute Value and ordering of rational numbers. | MGSE6.NS. 5 MGSE6.NS. 6 MGSE6.NS. 7 MGSE6.NS.7a MGSE6.NS.7b MGSE6.NS.7c MGSE6.NS.7d |
| **Planning A Field Trip (Spotlight Task) | Learning Task Individual/Partner | Absolute Value | MGSE6.NS. 5 <br> MGSE6.NS. 6 <br> MGSE6.NS. 7 <br> MGSE.NS.7a <br> MGSE6.NS.7b <br> MGSE6.NS.7c <br> MGSE6.NS.7d |

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In this task, students will plot integers and rational numbers on the number line. Students will order integers, determine distance between two points on a number line, work with negative and positive numbers as well as opposites.

## STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
MGSE6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3)=3$, and that 0 is its own opposite.
MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
MGSE6.NS. 7 Understand ordering and absolute value or rational numbers.

## STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about rational numbers with their visual representations. Students consider the value of these numbers in relation to distance (number lines).
3. Model with mathematics. Students use number lines to compare rational numbers.
4. Attend to precision. Students attend to the language of real-world situations to determine if positive or negative quantities/distances are being represented.

## ESSENTIAL QUESTIONS

- How do I use positive and negative numbers to represent quantities in real-world contexts?
- What are opposites and how are they shown on a number line?
- How can I use number lines to find the distances between points and compare/order rational numbers?


## MATERIALS

- colored pencils


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## **What's Your Sign?

## Part I: Representing numbers on a number line.

Directions: Use the thermometer to answer the questions. Use a blue colored pencil to represent colder temperatures, and use a red colored pencil to represent warmer temperatures.


1 A . Which temperature is colder, $-10^{\circ}$ or $0^{\circ}$ ?

## Solution

-10 degrees is colder than 0 degrees.
1B. Plot both numbers on the number line below.

## Solution

See number line.


2A. Which temperature is colder, $-5^{\circ}$ or $0^{\circ}$ ?
Solution
-5 degrees is colder than 0 degrees.
2B. Plot both numbers on the number line.

## Solution

See number line.


3 A . Which temperature is warmer, $-6^{\circ}$ or $-9^{\circ}$ ?

## Solution

-6 degrees is warmer than -9 degrees.

3B. Plot both numbers on the number line.

## Solution

See number line.


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4 A . Which temperature is warmer, $-2^{\circ}$ or $-5^{\circ}$ ?

## Solution

-2 degrees is warmer than -5 degrees.
4B. Plot both numbers on the number line.

## Solution

See number line.

5. What do you notice about negative numbers?

## Comment

Students should see the plots in blue, representing colder temperatures, are always to the left of the plots in red which represent warmer temperatures.

## Solution

Students should recognize all numbers below zero are located to the left of zero and all numbers above zero are located to the right of zero on the number line. Students should understand that when reading the number line from left to right, the numbers are ordered least to greatest; therefore, negative numbers are less than positive numbers and any positive number is greater than any negative number. Students should also understand the number to the left of any number is smaller in value. Consequently, the number to the right of any number is greater in value.

Comment
At this time, allow students to compare and order integers using multiple representations.

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## Part II: Absolute Value

Directions: Use the diagram of the city to answer the questions. Use a blue colored pencil to graph the locations on the number line.


1. If the park is located at zero on the number line, plot the location of the house and school if they are located one unit from the park. What do you notice about the placement of your plots on the number line?


## Solution

Both plots are one unit from zero. The distance between 0 and 1 is 1. The distance between -1 and 0 is also 1. There is one plot on positive one and one plot on negative one.

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2. Plot the location of the house and school if they are two units from the park. What do you notice about the placement of your plots on the number line?


## Solution

Both plots are two units from zero. The distance between 0 and 2 is 2 . The distance between -2 and 0 is also 2. There is one plot on positive two and one plot on negative two.
3. Plot the location of the house and school if they are five units from the park. What do you notice about the placement of your plots on the number line?


## Solution

Both plots are five units from zero. The distance between 0 and 5 is 5 . The distance between - 5 and 0 is also 5. There is one plot on positive five and one plot on negative five.
4. Plot the location of the house and school if they are nine units from the park. What do you notice about the placement of your plots on the number line?


## Solution

Both plots are nine units from zero. The distance between 0 and 9 is 9. The distance between 9 and 0 is also 9. There is one plot on positive nine and one plot on negative nine.

## Vocabulary Alert:

The distance between a number and zero on the number line is called absolute value. The symbol for absolute value is shown in this equation $|8|=8$ and $|-8|=8$. These are read as, "The absolute value of 8 equals 8 " and "the absolute value of negative 8 equals 8 ." This is true because the distance between 0 and 8 on the number line is 8 spaces and the distance between 0 and negative 8 on the number line is 8 spaces. Distance is always positive. One can never travel a negative distance.

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5. Explain |4|

## Solution

The absolute value of 4 is 4 because the distance between 0 and 4 is 4 .


## 6. Explain |-7|

## Solution

The absolute value of -7 is 7 because the distance between 0 and -7 is 7 .

7. Explain $|8|$

## Solution

The absolute value of 8 is 8 because the distance between 0 and 8 is 8 .

8. Explain $|-21|$

## Solution

The absolute value of -21 is 21 because the distance between 0 and -21 is 21 .

9. Explain $|d|$

## Solution

The absolute value of $d$ is $d$ because the distance between 0 and $d$ is $d$, where $d$ is any value.
10. Explain $|-d|$

## Solution

The absolute value of the opposite of $d(d)$ is $d$ because the distance between 0 and the opposite of $d(-d)$ is $d$, where $d$ is any value.

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## 11. Explain $\left|-\frac{1}{4}\right|$

## Solution

The absolute value of $-\frac{1}{4}$ is $\frac{1}{4}$ because the distance between 0 and $-\frac{1}{4}$ is $\frac{1}{4}$.
12. Explain $\left|\frac{3}{5}\right|$

## Solution

The absolute value of $\frac{3}{5}$ is $\frac{3}{5}$ because the distance between 0 and $\frac{3}{5}$ is $\frac{3}{5}$.

## 13. Explain |1.25|

## Solution

The absolute value of 1.25 is 1.25 because the distance between 0 and 1.25 is 1.25 .
14. Explain $|-5.6|$

## Solution

The absolute value of $\mathbf{- 5 . 6}$ is 5.6 because the distance between 0 and $\mathbf{- 5 . 6}$ is 5.6.
15. Explain $-|-8|$. This is read as, "The opposite of the absolute value of negative 8." Think about this. If the absolute value of negative 8 is eight, what is the opposite?

## Solution

The opposite of the absolute value of negative 8 is negative 8 because the distance between 0 and -8 is 8 and the opposite of 8 is negative 8 .
16. Explain $-|8|$. This is read as, "The opposite of the absolute value of 8 ." Think about this. If the absolute value of 8 is 8 , what is the opposite of 8 ?

## Solution

The opposite of the absolute value of 8 is negative 8 because the distance between 0 and 8 is 8 and the opposite of 8 is negative 8 .
17. Explain -|12|

## Solution

The opposite of the absolute value of 12 is negative 12 because the distance between 0 and 12 is 12 and the opposite of 12 is negative 12.

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## 18. Explain -|19|

## Solution

The opposite of the absolute value of 19 is negative 19 because the distance between 0 and 19 is 19 and the opposite of 19 is negative 19.
19. Explain $-|p|$

## Solution

The opposite of the absolute value of $p$ is "negative $p$ " because the distance between 0 and $p$ is $p$ and the opposite of $p$ is "negative $p$ ".
Variables should not be read as "negative p" because negative means the value is less than 0 and the value of $p$ is unknown.

## 20. Explain $-|-7|$

## Solution

The opposite of the absolute value of negative 7 is negative 7 because the distance between 0 and negative 7 is 7 and the opposite of 7 is negative 7.

## 21. Explain $-|-3|$

## Solution

The opposite of the absolute value of negative 3 is negative 3 because the distance between 0 and negative 3 is 3 and the opposite of 3 is negative 3.
22. Explain $-|-p|$

## Solution

The opposite of the absolute value of the opposite of $p$ is the opposite of $p$ because the distance between 0 and negative $p$ is $p$ and the opposite of $p$ is negative $p$.

Variables should not be read as "negative p" because negative means the value is less than 0 and the value of $p$ is unknown.

8-1P Note: Discussion of opposite numbers should occur at this time.

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## Part III: Opposites

Vocabulary Alert:
Opposite numbers are two different numbers that have the same absolute value. Example: 4 and -4 are opposite numbers because $|4|=4$ and $|-4|=4$.

## More about Opposite Numbers

1. When opposite numbers are added, the sum is zero.
2. To get the opposite of a number, change the sign.
3. The absolute values of opposite numbers are the same.
4. Opposite numbers are equidistant (the same distance) from 0 on a number line.

## Examples of Opposite Numbers

- 10 and -10 are the opposite numbers.
- -4 is the opposite number of 4 .
- 0 is the opposite of 0 .
- The opposite of negative 3 is 3

$$
\text { o Example: }-(-3)=3
$$

Directions: Use the number lines to find the opposite of the plotted point. Plot the opposite of the given number using a green colored pencil.

## 1.

## Solution

See number line

2.

## Solution

See number line


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## 3.

## Solution

See number line

4.

## Solution

See number line

5. Plot the opposite and write the decimal and fraction that are represented by the black and green dots.


## Solution

The number represented by the black dot is $-\frac{3}{5}$; the number represented by the green dot is $\frac{3}{5}$.
6. Plot the opposite and write the decimal and fraction that are represented by the black and green dots.


## Solution

The number represented by the black dot is $1 \frac{1}{5}$ and the number represented by the green dot is $-1 \frac{1}{5}$.

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7. Plot the opposite and write the decimal and fraction that are represented by the black and green dots.


## Solution

The number represented by the black dot is $-1 \frac{3}{5}$ and the number represented by the green dot is $1 \frac{3}{5}$.
8. Plot the opposite and write the decimal and fraction that are represented by the black and green dots.


Solution
The number represented by the black dot is $1 \frac{4}{5}$ and the number represented by the green dot is $-1 \frac{4}{5}$.
9. Plot $-(-7)$.

## Solution

See number line.

10. Plot $-\left(-1 \frac{1}{3}\right)$.

## Solution

See number line.

11. Plot $-\left(-1 \frac{1}{4}\right)$.

Solution
See number line.


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12. Plot the opposite of 1.75 .

## Solution

See number line.

13. Plot the opposite of 0.25 .

## Solution

See number line.

14. Plot the opposite of 0.20 .

## Solution

See number line.

15. Plot the opposite of -1.60 .

Solution
See number line.

16. Plot the opposite of ( -.25 ).

## Solution

See number line.


# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

Answer the following:
17. In a game of football, Jared gained 12 yards on the first play of the game. On the second play of the game, Jared lost 12 yards. How many total yards did Jared gain or lose?

## Solution

Jared did not gain or lose any yards. He is back where he started with the football. 12 and $\mathbf{- 1 2}$ are opposites. Therefore, the sum is zero.
18. Sydney Kate’s mom gave her $\$ 10$ for allowance. Sydney Kate owed her dad $\$ 10$ for the cool pair of socks that he purchased for her. How much money did Sydney Kate have left?

## Solution

Sydney Kate did not have any money left. 10 and -10 are opposites. Therefore, the sum is zero.
19. Brian has $\$ 60.42$ in his savings account. He really wants to purchase a volleyball net along with all the supplies to be able to have a game with his neighborhood friends. Brian spent $\$ 60.42$ on everything he needed. How much money does Brian now have in his savings account?

## Solution

Brian does not have any money in his savings account. $\$ 60.42$ and -60.42 are opposites. Therefore, the sum is zero.
20. Ciana is on a mountain top that is 18,240 feet above sea level. How far must she walk down the mountain to reach sea level?

## Solution

Ciana must walk 18,240 feet down the mountain to reach sea level. Sea level is represented by 0 on the number line. 18,240 and $-18,240$ are opposites. Therefore, the sum is zero.
21. What is the sum of -6 and 6 ?

## Solution

The sum of -6 and 6 is zero. These two numbers are opposites.

## **What's Your Sign?

## Part I: Representing numbers on a number line.

Directions: Use the thermometer to answer the questions. Use a blue colored pencil to represent colder temperatures, and use a red colored pencil to represent warmer temperatures.


1 A . Which temperature is colder, $-10^{\circ}$ or $0^{\circ} ?$

1B. Plot both numbers on the number line below.


2 A . Which temperature is colder, $-5^{\circ}$ or $0^{\circ}$ ?

2B. Plot both numbers on the number line.


3A. Which temperature is warmer, $-6^{\circ}$ or $-9^{\circ}$ ?

3B. Plot both numbers on the number line.


4 A . Which temperature is warmer, $-2^{\circ}$ or $-5^{\circ}$ ?

4B. Plot both numbers on the number line.

5. What do you notice about negative numbers?

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## Part II: Absolute Value

Directions: Use the diagram of the city to answer the questions. Use a blue colored pencil to graph the locations on the number line.


1. If the park is located at zero on the number line, plot the location of the house and school if they are located one unit from the park. What do you notice about the placement of your plots on the number line?

2. Plot the location of the house and school if they are two units from the park. What do you notice about the placement of your plots on the number line?


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3. Plot the location of the house and school if they are five units from the park. What do you notice about the placement of your plots on the number line?

4. Plot the location of the house and school if they are nine units from the park. What do you notice about the placement of your plots on the number line?


## Vocabulary Alert:

The distance between a number and zero on the number line is called absolute value. The symbol for absolute value is shown in this equation $|8|=8$ and $|-8|=8$. These are read as, "The absolute value of 8 equals 8 " and "the absolute value of negative 8 equals 8 ." This is true because the distance between 0 and 8 on the number line is 8 spaces and the distance between 0 and negative 8 on the number line is 8 spaces. Distance is always positive. One can never travel a negative distance.
5. Explain |4|.
6. Explain $|-7|$.
7. Explain |8|.

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8. Explain $|-21|$.
9. Explain $|d|$.
10. Explain $|-d|$.
11. Explain $\left|-\frac{1}{4}\right|$.
12. Explain $\left|\frac{3}{5}\right|$.
13. Explain |1.25|.
14. Explain $|-5.6|$.
15. Explain $-|-8|$. This is read as, "The opposite of the absolute value of negative 8." Think about this. If the absolute value of negative 8 is eight, what is the opposite?
16. Explain $-|8|$. This is read as, "The opposite of the absolute value of 8 ." Think about this. If the absolute value of 8 is 8 , what is the opposite?
17. Explain -|12|.
18. Explain -|19|.
19. Explain $-|p|$.
20. Explain $-|-7|$.
21. Explain $-|-3|$.
22. Explain $-|-p|$.

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## Part III: Opposites

## Vocabulary Alert:

Opposite numbers are two different numbers that have the same absolute value. Example: 4 and -4 are opposite numbers because $|4|=4$ and $|-4|=4$.

## More about Opposite Numbers

1. When opposite numbers are added, the sum is zero.
2. To get the opposite of a number, change the sign.
3. The absolute values of opposite numbers are the same.
4. Opposite numbers are equidistant from 0 on a number line.

## Examples of Opposite Numbers

- 10 and -10 are the opposite numbers.
- -4 is the opposite number of 4 .
- 0 is the opposite of 0 .
- The opposite of negative 3 is 3

$$
\text { o Example: }-(-3)=3
$$

Directions: Use the number lines to find the opposite of the plotted point. Plot the opposite of the given number using a green colored pencil.
1.

2.

3.

4.


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5. Plot the opposite and tell the decimal and fraction that are represented by the black and green dot.

6. Plot the opposite and tell the decimal and fraction that are represented by the black and green dot.

7. Plot the opposite and tell the decimal and fraction that are represented by the black and green dot.

8. Plot the opposite and tell the decimal and fraction that are represented by the black and green dot.

9. Plot $-(-7)$.

10. Plot $-\left(-1 \frac{1}{3}\right)$.


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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
11. Plot $-\left(-1 \frac{1}{4}\right)$.

12. Plot the opposite of 1.75 .

13. Plot the opposite of 0.25 .

14. Plot the opposite of 0.20 .

15. Plot the opposite of -1.60 .

16. Plot the opposite of (-.25).


# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

Answer the following:
17. In a game of football, Jared gained 12 yards on the first play of the game. On the second play of the game, Jared lost 12 yards. How many total yards did Jared gain or lose?
18. Sydney Kate’s mom gave her $\$ 10$ for allowance. Sydney Kate owed her dad $\$ 10$ for the cool pair of socks that he purchased for her. How much money did Sydney Kate have left?
19. Brian has $\$ 60.42$ in his savings account. He really wants to purchase a volleyball net along with all the supplies to be able to have a game with his neighborhood friends. Brian spent $\$ 60.42$ on everything he needed. How much money does Brian now have in his savings account?
20. Ciana is on a mountain top that is 18,240 feet above sea level. How far must she walk down the mountain to reach sea level?
21. What is the sum of -6 and 6 ?

In this task, students will convert among different forms of rational numbers, compare sizes of numbers in the same form, and use the idea that the absolute value of a number is the distance between the number and 0 to order rational numbers on a number line. Students also use a number line to model opposites.

## STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS. 5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
MGSE6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3)=3$, and that 0 is its own opposite.
MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
MGSE6.NS. 7 Understand ordering and absolute value or rational numbers.

## STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about rational numbers with their visual representations. Students consider the value of these numbers in relation to distance (number lines).
3. Construct viable arguments and critique the reasoning of others. Students construct arguments regarding number line representations

## ESSENTIAL QUESTIONS

- Why is it useful for me to know the absolute value of a number?
- What are opposites and what characteristics do they have on the number line?
- How do I compare and order rational numbers?


## MATERIALS

Pencil
Paper

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Representing Rational Numbers on the Number Line

Below is a number line from -1 to 1 .

A. On the number line, mark $0,-\frac{2}{5},-\frac{1}{3}, \frac{1}{2}, \frac{1}{5}, \frac{1}{4},-0.8$.

## Solution

See number line above
B. Choose one fraction and one decimal that are between $\frac{1}{5}$ and $\frac{1}{4}$ on the number line. Your fractions and your decimal should not be equivalent. Plot these on the number line below. Below is a number line from -1 to 1 .


## Comment

The fraction should be between $\frac{1}{5}$ and $\frac{1}{4}$ and the decimal should be between 0.20 and 0.25 .
C. Explain how you know each of your answers is between $\frac{1}{5}$ and $\frac{1}{4}$.

## Solution

The students should be able to explain their answers by comparing fractions or decimals.
D. Plot two additional points that are opposites on the number line below. How do you know that these two points are opposites?


## Comment

The plotted points should be equidistant from zero on the number line. Students should explain that opposites are two numbers that are equidistant from zero on the number line. Furthermore, students may explain that the sum of two opposites is zero.

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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Representing Rational Numbers on the Number Line

Below is a number line from -1 to 1 .

A. On the number line, mark $0,-\frac{2}{5}, 5,-\frac{1}{3}, \frac{1}{2}, \frac{1}{5}, \frac{1}{4},-0.8$
B. Choose one fraction and one decimal that are between $\frac{1}{5}$ and $\frac{1}{4}$ on the number line. Your fractions and your decimal should not be equivalent. Plot these on the number line below. Below is a number line from -1 to 1 .

C. Explain how you know each of your answers is between $\frac{1}{5}$ and $\frac{1}{4}$.
D. Plot two additional points that are opposites on the number line below. How do you know that these two points are opposites?


## Symbols of Inequalities and the Coordinate System

In this task, students will plot points on a number line and a coordinate grid. Students will also use absolute value to determine distance between two points. In addition, students will write inequality statements to show the relationship between two numbers.

## STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
MGSE6.NS. 7 Understand ordering and absolute value or rational numbers.
MGSE6NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
MGSE6.NS.7b Write, interpret, and explain statements of order for rational numbers in realworld contexts.
MGSE6.NS. 8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of problems involving points in the coordinate plane.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about rational numbers with their visual representations. Students consider the value of these numbers in relation to distance (number lines).
3. Construct viable arguments and critique the reasoning of others. Students construct arguments regarding number line representations.
4. Model Mathematics. Students use number lines to compare number and represent inequalities in mathematical and real-world contexts.
5. Use appropriate tools strategically. Students use number line to represent situations involving positive and negative numbers.
6. Attend to precision. Students attend to the language of real-world situations to determine if positive or negative quantities/distances are being represented.
7. Look for and make use of structure. Recognizing that the axes on the coordinate plane are number lines.

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8. Look for and express regularity in repeated reasoning. Students relate new experiences to experiences with similar context when studying positive and negative representations of distance and quantity.

## ESSENTIAL QUESTIONS

- When are negative numbers used and why are they important?
- Why is it useful for me to know the absolute value of a number?
- How do I plot points on the coordinate plane?
- How do I write inequalities using the > and < symbol?
- How is the absolute value of a number used to determine its distance on the coordinate plane?
- How can I use vertical and horizontal number lines to solve problems?
- How do I compare and order rational numbers.


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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Learning Task: Symbols of Inequalities and the Coordinate System

As a middle school student, you are very familiar with the equal sign. You know that in order for two numbers to be equal, they must have the same value. Mathematicians also deal with symbols of inequality such as greater than ( $>$ ) and less than( $<$ ). These two symbols of inequality can help us compare rational numbers.

1. A grocery store is located in the center of the town at the right. The coordinates of the store are $(0,0)$, and the store is represented by a star. Mary lives 3 blocks east of the grocery store and Julia lives 5 blocks west of the grocery store. Plot the location of each house, and give the coordinates of Mary's house and Julie's house. How far does each girl live from the store? Who lives a greater distance from the store? Write two inequality statements to compare the distance that each girl lives from the grocery store.


## Solution

There should be a point located at $(0,3)$ to represent


Mary's house and a point located at (0, -5) to represent Julia's house. Mary lives 3 blocks from the store and Julia lives five blocks from the store. Julia lives the greater distance from the store. The inequality statements are $5>3$ and $3<5$.
2. A grocery store is located in the center of the town at the right. The coordinates of the store are $(0,0)$, and the store is represented by a star. Musa lives 3 blocks east of the grocery store and Andre lives 5 blocks east of the grocery store. Plot the location of each house, and give the coordinates of Musa's house and Andre's house. How far does each boy live from the store? Who lives a greater distance from the store? Write two inequality statements to compare the distance that each boy lives from the grocery store.

## Solution

There should be a point located at $(0,3)$ to represent
 Musa's house and a point located at $(0,5)$ to represent Andre's house. Musa lives 3 blocks from the grocery store and Andre lives five blocks from the grocery store. Andre lives a greater distance from the store. The inequality statements are $5>3$ and $3<5$.

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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
3. A grocery store is located in the center of the town at the right. The coordinates of the store are $(0,0)$, and the store is represented by a star. Madison lives three blocks west of the grocery store and Gavin lives 5 blocks west of the grocery store. Plot the location of each house, and give the coordinates of Madison's house and Gavin’s house. How far does each person live from the store? Who lives a greater distance from the store? Write two inequality statements to compare the distance that each person lives from the grocery store.

## Solution

There should be a point located at $(0,-3)$ to represent Madison's house and a point located at $(0,-5)$ to represent Gavin's house. Madison lives 3 blocks from the grocery store and Gavin lives five blocks from the grocery store. Gavin lives a greater distance from the store. The inequality statements are $5>3$ and $3<5$.

4. There are two thermometers at the right. Write two inequality statements to describe the relationship between the two thermometers.

## Solution

The inequality statements are $60>20$ and $20<60$.


# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
5. There are two thermometers at the right. Write two inequality statements to describe the relationship between the two thermometers.

## Solution

The inequality statements are $-3>-7$ and $-7<-3$.
6. Below is a chart showing the average depth of many of the world's oceans.

| Depth's of the World's Oceans |  |
| :---: | :---: |
| OCEAN | Average Depth (feet) |
| Pacific Ocean | 15200 feet |
| Atlantic Ocean | 12900 feet |
| Indian Ocean | 13000 feet |
| Arctic Ocean | 4000 feet |


A. Draw a vertical number line with 0 as sea level to represent the depth of the oceans. Be careful when determining your scale and interval.

B. Write two inequality statements to compare the following:

1. The relationship between the Pacific Ocean and the Atlantic Ocean.

## Solution

The inequality statements are $-12900>-15200$ and $-15200<-12900$.
2. The relationship between the Indian and the Arctic Ocean.

Solution
The inequality statements are $-4000>-13000$ and $-13000<-4000$.
3. The relationship between the Atlantic Ocean and the Indian Ocean.

## Solution

The inequality statements are $-12900>-13000$ and $-13000<-12900$.
4. The relationship between the Pacific Ocean and the Arctic Ocean.

## Solution

The inequality statements are $-4000>-15200$ and $-15200<-4000$.
7. Brian has a checkbook balance of $-\$ 23.45$, and Kristen has a checkbook balance of -18.42 . Write a statement of inequality to represent the situation. Who owes more money, Brian or Kristen?

## Solution

The inequality statements are $-18.42>-23.45$ and $-23.45<-18.42$. Brian owes more money than Kristen and Kristen owes less money than Brian.
8. Robin and Trey love the cold weather. In fact, the colder the weather, the happier Robin and Trey are. Both Robin and Trey have very precise thermometers that measure the temperature to the nearest tenth of a degree. The temperature in Robin's location is $-6.8^{\circ} \mathrm{C}$ and the weather in Trey's location is $-14.2^{\circ} \mathrm{C}$. Trey tells Robin that his location is warmer than her location because 14.2 is greater than 6.8, but Robin disagrees with Trey. Plot the two temperatures on a number line. Then write two statements of inequality to represent the situation. Is Robin or Trey colder?

## Solution

The inequality statements are $-6.8>-14.2$ and $-14.2<-6.8$. Trey's location is colder than Robin's location or Robin's location is warmer than Trey's location.

9. Lynn and Max are divers. Lynn's depth gauge says she is 43.6 feet deep. Max's depth gauge says that he is 54.2 feet deep. Who is closer to the surface? Write two statements of inequality to describe the situation.

## Solution

The inequality statements are $-43.6>-54.2$ and $-54.2<-43.6$. Therefore, Lynn is closer to the surface.

## Georgia Department of Education

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Learning Task: Symbols of Inequalities and the Coordinate System

As a middle school student, you are very familiar with the equal sign. You know that in order for two numbers to be equal, they must have the same value. Mathematicians also deal with symbols of inequality such as greater than ( $>$ ) and less than ( $<$ ). These two symbols of inequality can help us compare rational numbers.
Solve the following problems on the coordinate plane or number line. Use the coordinate plane or number line to help you explain your answer. You should also write a statement of inequality to justify your answer.

1. A grocery store is located in the center of the town at the right. The coordinates of the store are $(0,0)$, and the store is represented by a star. Mary lives 3 blocks east of the grocery store and Julia lives 5 blocks west of the grocery store. Plot the location of each house, and give the coordinates of Mary's house and Julie's house. How far does each girl live from the store? Who lives a greater distance from the store? Write two inequality statements to compare the distance that each girl lives from the grocery store.

2. A grocery store is located in the center of the town at the right. The coordinates of the store are $(0,0)$, and the store is represented by a star. Musa lives 3 blocks east of the grocery store and Andre lives 5 blocks east of the grocery store. Plot the location of each house, and give the coordinates of Musa's house and Andre's house. How far does each boy live from the store? Who lives a greater distance from the store? Write two inequality statements to compare the distance that each boy lives from the grocery store.


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3. A grocery store is located in the center of the town at the right. The coordinates of the store are $(0,0)$, and the store is represented by a star. Madison lives three blocks west of the grocery store and Gavin lives 5 blocks west of the grocery store. Plot the location of each house, and give the coordinates of Madison's house and Gavin's house. How far does each person live from the store? Who lives a greater distance from the store? Write two inequality statements to compare the distance that each person lives from the grocery store.

4. There are two thermometers at the right. Write two inequality statements to describe the relationship between the two thermometers.

5. There are two thermometers at the right. Write two inequality statements to describe the relationship between the two thermometers.


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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
6. Below is a chart showing the average depth of many of the world's oceans.

| Depth's of the World's Oceans |  |
| :---: | :---: |
| OCEAN | Average Depth (feet) |
| Pacific Ocean | 15200 feet |
| Atlantic Ocean | 12900 feet |
| Indian Ocean | 13000 feet |
| Arctic Ocean | 4000 feet |

A. Draw a vertical number line with 0 as sea level to represent the depth of the oceans. Be careful when determining your scale and interval.
B. Write two inequality statements to compare the following:

1. The relationship between the Pacific Ocean and the Atlantic Ocean.
2. The relationship between the Indian and the Arctic Ocean.
3. The relationship between the Atlantic Ocean and the Indian Ocean.
4. The relationship between the Pacific Ocean and the Arctic Ocean.

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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
7. Brian has a checkbook balance of $-\$ 23.45$, and Kristen has a checkbook balance of -18.42 . Write a statement of inequality to represent the situation. Who owes more money, Brian or Kristen?
8. Robin and Trey love the cold weather. In fact, the colder the weather, the happier Robin and Trey are. Both Robin and Trey have very precise thermometers that measure the temperature to the nearest tenth of a degree. The temperature in Robin's location is $-6.8^{\circ} \mathrm{C}$ and the weather in Trey's location is $-14.2^{\circ} \mathrm{C}$. Trey tells Robin that he is warmer than she is because 14.2 is greater than 6.8, but Robin disagrees with Trey. Plot the two temperatures on a number line. Then write two statements of inequality to represent the situation. Is Robin or Trey colder?
9. Lynn and Max are divers. Lynn's depth gauge says she is 43.6 feet deep. Max's depth gauge says that he is 54.2 feet deep. Who is closer to the surface? Write two statements of inequality to describe the situation.

In this task, students move their thinking about distance from the horizontal number line and a vertical number line to the coordinate plane. They plot points, reflect over the $x$-axis or the $y$ axis, use opposites and absolute value to determine distances, identify and draw polygons, find the length of the sides of the polygons, and apply these techniques to the context of solving realworld mathematical problems.

## STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
MGSE6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3)=3$, and that 0 is its own opposite.
MGSE6.NS.6b Understand signs of number in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
MGSE6.NS. 7 Understand ordering and absolute value or rational numbers.
MGSE6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a realworld situation.
MGSE6.NS. 8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
MGSE6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply those techniques in the context of solving real-world mathematical problems.

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of problems involving points and polygons in the coordinate plane.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about rational numbers with their visual representations. Students consider the value of these numbers in relation to distance (coordinate plane).

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3. Construct viable arguments and critique the reasoning of others. Students construct arguments regarding number line representations
4. Model with mathematics. Students use the coordinate plane to reflect points across the axes.
5. Use appropriate tools strategically. Students use number lines to represent situations involving positive and negative numbers.
6. Attend to precision. Students attend to the language of real-world situations to determine if positive or negative quantities/distances are being represented.
7. Look for and make use of structure. Students recognize that the axes on the coordinate plane are number lines.
8. Look for and express regularity in repeated reasoning. Students relate new experiences to experiences with similar context when studying positive and negative representations of distance and quantity.

## ESSENTIAL QUESTIONS

- How are opposites and absolute value related?
- How can I use absolute value to help me determine the distance between two points on a coordinate plane?
- What generalizations can I make about reflecting points across the $x$-axis or the $y$-axis?


## Graphing on the Coordinate Plane

Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on the number line extends to graphing and reflecting points across the $x$-axes (horizontal number line) or the $y$-axis (vertical number line) on a coordinate plane.

1. On the horizontal number line, plot 7 and -7 . What is the distance of each point from zero? What is the distance between 7 and -7 ? How does absolute value help you write a number sentence to help you find the distance between 7 and -7 ?


## Solution

Each point is 7 units from 0. The distance between 7 and -7 is 14 units. Absolute value helps us write a number sentence to help find the distance between 7 and -7 by using the distance from 0 on the number line. $|7|+|-7|=14$.
2. On the horizontal number line, plot 5 and -5 . What is the distance of each point from zero? What is the distance between 5 and -5 ? How does absolute value help you write a number sentence to help you find the distance between 5 and -5 ?


## Solution

Each point is 5 units from 0. The distance between 5 and -5 is 10 units. Absolute value helps us write a number sentence to help find the distance between 5 and -5 by using the distance from 0 on the number line. $|5|+|-5|=10$.

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3. On the horizontal number line, plot 2 and -2 . What is the distance of each point from zero? What is the distance between 2 and -2 ? How does absolute value help you write a number sentence to help you find the distance between 2 and -2 ?


## Solution

Each point is 2 units from 0. The distance between 2 and -2 is 4 units. Absolute value helps us write a number sentence to help find the distance between 2 and -2 by using the distance from 0 on the number line. $|2|+|-2|=4$.
4. On the vertical number line, plot 1 and -1 . What is the distance of each point from zero? What is the distance between 1 and -1 ? How does absolute value help you write a number sentence to help you find the distance between 1 and -1 ?


## Solution

Each point is 1 unit from 0. The distance between 1 and -1 is 2 units. Absolute value helps us write a number sentence to help find the distance between 1 and -1 by using the distance from 0 on the number line. $|1|+|-1|=2$.

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5. On the vertical number line, plot 10 and -10 . What is the distance of each point from zero? What is the distance between 10 and -10 ? How does absolute value help you write a number sentence to help you find the distance between 10 and -10 ?


## Solution

Each point is 10 units from 0. The distance between 10 and -10 is 20 units. Absolute value helps us write a number sentence to help find the distance between 10 and -10 by using the distance from 0 on the number line. $|10|+|-10|=20$.
6. On the vertical number line, plot 8 and -8 . What is the distance of each point from zero? What is the distance between 8 and -8 ? How does absolute value help you write a number sentence to help you find the distance between 8 and -8 ?


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## Solution

Each point is 8 units from 0. The distance between 8 and -8 is 16 units. Absolute value helps us write a number sentence to help find the distance between 8 and -8 by using the distance from 0 on the number line. $|8|+|-8|=16$.
7. The points $(1,3),(-1,5),(-3,3)$, and $(4,-4)$ have been graphed on the coordinate plane. Reflect each point across the $x$-axis. What are the coordinates of the reflected points?

When the star $(1,3)$ is reflected across the $x$-axis, the new point is located at $\qquad$ -

## Solution <br> (1,-3)

When the triangle $(-1,5)$ is reflected across the $x$ -
 axis, the new point is located at $\qquad$ _.

## Solution

(-1, -5)
When the smiley face $(-3,3)$ is reflected across the $x$-axis, the new point is located at $\qquad$ .

## Solution

(-3, -3)
When the lightning bolt $(4,-4)$ is reflected across the $x$-axis, the new point is located at $\qquad$ .

## Solution

$(4,4)$
What similarities do you notice between the coordinates of the original point and the reflected point?

## Solution

The x coordinate stays the same. The y coordinate becomes the opposite.

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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • I Tnit7
8. The points $(1,3),(-1,5),(-3,3)$, and $(4,-4)$ have been graphed on the coordinate plane. Reflect each point across the $y$-axis. What are the coordinates of the reflected points?

When the star $(1,3)$ is reflected across the $y$-axis, the new point is located at $\qquad$ .

## Solution

$(-1,3)$
When the triangle $(-1,5)$ is reflected across the $y$ axis, the new point is located at $\qquad$ .


## Solution

$(1,5)$
When the smiley face $(-3,3)$ is reflected across the $y$-axis, the new point is located at $\qquad$ .

## Solution

$(3,3)$
When the lightning bolt $(4,-4)$ is reflected across the $y$-axis, the new point is located at $\qquad$ .

## Solution

(-4, -4)
What similarities do you notice between the coordinates of the original point and the reflected point?

## Solution

The $x$ coordinate becomes the opposite. The y coordinate stays the same.

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9. The smiley face, located at point $(-4,5)$, has been reflected across the $y$-axis. The new location of the smiley face is $(4,5)$. What is the distance between $(-4,5)$ and $(4,5)$ ? Write a number sentence using the distance from the $y$-axis to help justify your answer.

## Solution

The distance between the two points is 8 units. $|-4|+|4|=8$

10. A point, located at ( $-3,-4$ ), has been reflected across the $x$-axis. The new point has the coordinates $(-3,4)$. What is the distance between $(-3,-4)$ and $(-3,4)$ ? Write a number sentence using the distance from the $x$-axis to help justify your answer.

Solution
$|-4|+|4|=8$


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Use the drawing of the city to help you answer questions 11-17.

11. What is the location of city hall? What is the location of the police station? How many blocks apart are these two buildings?

## Solution

City hall is located at (0, 0), and the police station is located at (0, -4). The two buildings are 4 blocks apart.
12. What is the location of the art museum? What is the location of the animal shelter? How many blocks apart are these two buildings?

## Solution

The art museum is located at (6, 1), and the animal shelter is located at (6, -2). The two buildings are 3 blocks apart.
13. What is the location of the hospital? What is the location of the cemetery? How many blocks apart are these two buildings?

## Solution

The hospital is located at (-6, -4), and the cemetery is located at (3, -4). The two buildings are 9 blocks apart.
14. What is the location of the hospital? What is the location of the police station? How many blocks apart are these two buildings?

## Solution

The hospital is located at (6, -4), and the police station is located at (0, -4). The two buildings are 6 blocks away.

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15. The police station is being moved to its new location located at $(-6,-1)$. Is the police station closer to, farther away from, or the same distance from the Hospital?
Solution
The police station is now three blocks away from the hospital. It was six blocks away. It is now closer to the hospital.
16. The art museum and the animal shelter are moving as well. Their movement can be described as a reflection across the $y$-axis. What are the coordinates of the new location for the art museum and the animal shelter? How many blocks are they from each other? Is this the same distance as in question 12? Why or why not?

## Solution

The art museum is now located at $(-6,1)$, and the animal shelter is now located at $(-6,-2)$. These two buildings are three blocks apart. This is the same distance as they were in question 12 when they were located at $(6,1)$ and $(6,-2)$. The reason they are the same distance apart is because when they are reflected across the $y$-axis the $y$ coordinate stays the same and the $x$ coordinate becomes the opposite. This keeps the two buildings the same distance apart.
17. The stadium is also being moved. Its new location can be described as a reflection across the $x$-axis. What is the new location of the stadium? How many blocks is the new stadium from the old stadium?

## Solution

The stadium is now located at (-2, -3). The stadium is now 6 blocks from the old stadium.
18. On a map, the library is located at $(-2,2)$, the city hall is located at $(0,2)$, and the middle school is located at $(0,0)$.
A. Represent the locations as points on a coordinate grid with a unit of 1 mile.

## Solution

See Grid on left.
B. What shape is formed by connecting the three locations?


## Solution

A triangle is formed by connecting the three locations.

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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
C. The city council is planning to place a city park in this area. How large is the area of the planned park?

## Solution

The area of the planned park is 2 square miles. The formula $A=\frac{1}{2} b h$ or $A=\frac{b h}{2}$ can be used to solve this. The base is $\mathbf{2}$ miles and the height is $\mathbf{2}$ miles.
19. On the map, the elementary school is located at $(-4,2)$, the middle school is located at $(2,2)$, and, the high school is located at (-4, -3 ).
A. Each interval on the number lines represents two miles ( 0 to 1 represents 2 miles). Each school forms the vertex of a rectangle. If the district office for the school system is the fourth vertex of the rectangle, what are the coordinates? How do you know?

## Solution



The district office would be located at (2, -3). I know this because the point where the high school is located would be reflected across the $y$-axis or the point where the middle school is would be reflected across the x-axis.
B. What are the length and width of the rectangle?

## Solution

The length of the rectangle is 12 miles. The width of the rectangle is 10 miles.
C. What is the perimeter of the rectangle?

## Solution

The perimeter of the rectangle, found by using the formula $P=2 l+2 w$ or $P=l+l+w+$ $w$, is 44 miles.
D. What is the area of the rectangle?

## Solution

The area of the rectangle, found by using the formula $A=l w$ is 120 square miles.

## Graphing on the Coordinate Plane

Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on the number line extends to graphing and reflecting points across the $x$-axes (horizontal number line) or the $y$-axis (vertical number line) on a coordinate plane.

1. On the horizontal number line, plot 7 and -7 . What is the distance of each point from zero? What is the distance between 7 and -7 ? How does absolute value help you write a number sentence to help you find the distance between 7 and -7 ?

2. On the horizontal number line, plot 5 and -5 . What is the distance of each point from zero? What is the distance between 5 and -5 ? How does absolute value help you write a number sentence to help you find the distance between 5 and -5 ?

3. On the horizontal number line, plot 2 and -2 . What is the distance of each point from zero? What is the distance between 2 and -2 ? How does absolute value help you write a number sentence to help you find the distance between 2 and -2 ?

4. On the vertical number line, plot 1 and -1 . What is the distance of each point from zero? What is the distance between 1 and -1 ? How does absolute value help you write a number sentence to help you find the distance between 1 and -1 ?

5. On the vertical number line, plot 10 and -10 . What is the distance of each point from zero? What is the distance between 10 and -10 ? How does absolute value help you write a number sentence to help you find the distance between 10 and -10 ?


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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
6. On the vertical number line, plot 8 and -8 . What is the distance of each point from zero? What is the distance between 8 and -8 ? How does absolute value help you write a number sentence to help you find the distance between 8 and -8 ?

7. The points $(1,3),(-1,5),(-3,3)$, and $(4,-4)$ have been graphed on the coordinate plane. Reflect each point across the $x$-axis. What are the coordinates of the reflected points?

When the star $(1,3)$ is reflected across the $x$-axis, the new point is located at $\qquad$ .

When the triangle $(-1,5)$ is reflected across the $x$ axis, the new point is located at $\qquad$ .

When the smiley face $(-3,3)$ is reflected across the $x$ axis, the new point is located at $\qquad$ .


When the lightning bolt (4, -4 ) is reflected across the $x$-axis, the new point is located at $\qquad$ .

What similarities do you notice between the coordinates of the original point and the reflected point?

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
8. The points $(1,3),(-1,5),(-3,3)$, and $(4,-4)$ have been graphed on the coordinate plane. Reflect each point across the $y$-axis. What are the coordinates of the reflected points?

When the star $(1,3)$ is reflected across the $y$-axis, the new point is located at $\qquad$ .

When the triangle $(-1,5)$ is reflected across the $y$-axis, the new point is located at $\qquad$ .

When the smiley face $(-3,3)$ is reflected across the $y$ axis, the new point is located at $\qquad$ .

When the lightning bolt ( $4,-4$ ) is reflected across the $y$-axis, the new point is located at $\qquad$ —.

What similarities do you notice between the coordinates of the original point and the reflected point?’

9. The smiley face, located at point $(-4,5)$, has been reflected across the $y$-axis. The new location of the smiley face is $(4,5)$. What is the distance between $(-4,5)$ and $(4,5)$ ? Write a number sentence using the distance from the $y$-axis to help justify your answer.


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10. A point, located at $(-3,-4)$, has been reflected across the $x$-axis. The new point has the coordinates $(-3,4)$. What is the distance between $(-3,-4)$ and $(-3,4)$ ? Write a number sentence using the distance from the $x$-axis to help justify your answer.

Use the drawing of the city to help you answer questions 11-17.


11. What is the location of city hall? What is the location of the police station? How many blocks apart are these two buildings?
12. What is the location of the art museum? What is the location of the animal shelter? How many blocks apart are these two buildings?
13. What is the location of the hospital? What is the location of the cemetery? How many blocks apart are these two buildings?
14. What is the location of the hospital? What is the location of the police station? How many blocks apart are these two buildings?

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15. The police station is being moved to its new location located at $(-6,-1)$. Is the police station closer to, farther away from, or the same distance from the Hospital?
16. The art museum and the animal shelter are moving as well. Their movement can be described as a reflection across the $y$-axis. What are the coordinates of the new location for the art museum and the animal shelter? How many blocks are they from each other? Is this the same distance as in question 12? Why or why not?
17. The stadium is also being moved. Its new location can be described as a reflection across the $x$-axis. What is the new location of the stadium? How many blocks is the new stadium from the old stadium?
18. On a map, the library is located at $(-2,2)$, the city hall is located at $(0,2)$, and the middle school is located at $(0,0)$.
A. Represent the locations as points on a coordinate grid with a unit of 1 mile.
B. What shape is formed by connecting the three locations?
C. The city council is planning to place a city park in this area. How large is the area of the planned park?

19. On the map, represent the locations as points on the coordinate plane. The elementary school is located at $(-4,2)$, the middle school is located at $(2,2)$, and, the high school is located at ( $-4,-3$ ).
A. Each interval on the number lines represents 2 miles ( 0 to 1 represents 2 miles). Each school forms the vertex of a rectangle. If the district office for the school system is the fourth vertex of the rectangle, what are the coordinates? How do you know?


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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
B. What are the length and width of the rectangle?
C. What is the perimeter of the rectangle?
D. What is the area of the rectangle?

## Formative Assessment Lesson: Integers on the Coordinate Plane Back to Task Table

Source: Georgia Mathematics Design Collaborative
This lesson is intended to help you assess how well students are able to:

- Plot points in the four quadrants of the coordinate plane
- Work with reflections in the coordinate plane


## STANDARDS ADDRESSED IN THIS TASK:

MGSE6.NS.6b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

## STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:
2. Reason abstractly and quantitatively.
7. Look for and make use of structure.
8. Look for/express regularity in repeated reasoning.

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, Integers on the Coordinate Plane, is a Formative Assessment Lesson (FAL) that can be found at: http://ccgpsmathematics6-
$\underline{\text { 8.wikispaces.com/Georgia+Mathematics+Design+Collaborative }+ \text { Formative }+ \text { Assessment }+ \text { Lesso } ~}$ ns

## Absolute Value and Ordering

Back to Task Table

In this task, students will distinguish between absolute value and order.
This task has two versions below. The first version has the answers in the teacher's version and is scaffolded.

## STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS. 5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
MGSE6.NS. 7 Understand ordering and absolute value of rational numbers.
MGSE6NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
MGSE6.NS.7b Write, interpret, and explain statements of order for rational numbers in realworld contexts.
MGSE6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a realworld situation.
MGSE6.NS.7d Distinguish comparisons of absolute value from statements about order.

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about rational numbers with their visual representations. Students consider the values and absolute values of numbers in relation to number line
2. Construct viable arguments and critique the reasoning of others. Students construct statements to explain the order of numbers and the order of their absolute values.
3. Model with mathematics. Students use number lines and absolute values to compare numbers in a real-world context.
4. Use appropriate tools strategically. Students use number lines to represent situations involving positive and negative numbers.
5. Attend to precision. Students attend to the language of real-world situations to determine if positive or negative quantities/distances are being represented.
6. Look for and express regularity in repeated reasoning. Students relate new experiences to experiences with similar context when studying positive and negative representations of order and absolute value

## ESSENTIAL QUESTIONS

- How is absolute value related to order?


## TASK COMMENTS

For negative numbers, as the Absolute Value increases, the value of the number decreases. The greater the debt, the less money you have. The greater the depth, the farther the location is from the surface.

## Links for instructional resources and lesson plans for this mathematical concept:

http://learnzillion.com/lessons/1144-use-a-number-line-to-understand-how-while-the-value-of-a-negative-number-decreases-its-absolute-value-increases
http://learnzillion.com/lessons?utf8=\�\�\�\&filters\[subject\]=math\&query=\&filt ers\%5Bgrade\%5D\%5B\%5D=6\&filters\%5Bdomain\%5D=\&filters\%5Bstandard\%5D

## Version 1- Scaffolded Task

## Absolute Value and Ordering

For negative numbers, as the absolute value increases the value of the number decreases. The greater the debt the less money you have. The greater the depth the farther away from the surface.
1.
A. A mother dolphin is 150.25 meters below sea level. Her calf is 45 meters below sea level. Which dolphin is farthest from the surface?
Solution:

## The mother

B. Draw this situation on a vertical number line? Does this illustrate absolute value or ordering (for ordering use <, >)? Justify your answer.

Student number lines will vary according to intervals chosen by the students. Check for student understanding of intervals. This statement deals with absolute value if the student's statement deals with the distance from 0 or the surface. It deals with order if they are comparing that one is deeper or farther than the other.

Ex. Order $=-150.25<-45$ or $-45>-150.25$
the baby calf is closer to the surface than the mother. $\mathbf{- 1 5 0 . 2 5}$ is less than -45 and represents a greater depth.

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Ex. Absolute Value = Statements about distance from the surface. Ex. The baby has less depth and greater value. On the negative side - the greater the absolute value the less value the number has.
C. A mother whale is at 35 meters below the surface and her calf is at the surface. How far does the calf have to swim to get to its mother?

## Solution:

## 35 meters

D. Draw this situation on a vertical number line? Does this illustrate absolute value or ordering (for ordering use <, >) ? Justify your answer.

Student number lines will vary according to intervals chosen by the students. Check for student understanding of intervals.

Ex. Order $=-35<0$ or $0>-35$
-35 is less than 0 and represents a depth of 35 ft below 0
Ex. Absolute Value = Statements about distance from the surface. Ex. The baby has less depth and greater absolute value. On the negative side - the greater the absolute value the less value the number has. The mother is at 35 ft below 0 which is an absolute value of 35 and the baby has an absolute value of 0 .
2. Tonya, Beverly, Ashley, Janet, and Anne want to figure out who has the most debt and who has the most money.

Tonya - $\$ 5.75$
Beverly $\$ 12$
Janet \$6.25
Anne -\$4.15
Ashley -\$3
A. Graph the five numbers on the number line.

Student number lines will vary according to intervals chosen by the students. Check for student understanding of intervals.
B. Put the numbers in order

Solution:
$-5.75,-4.15,-3,6.25,12$

Mathematics • Grade 6 • Unit 7: Rational Explorations: Numbers and their Opposites Richard Woods, State School Superintendent July 2015 • Page 69 of 97 All Rights Reserved

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C. What is the absolute value of each term in the list? Graph the absolute value of each term on a horizontal number line.

## Solution:

$$
5.75,12,6.25,4.15,3
$$

Student number lines will vary according to intervals chosen by the students. Check for student understanding of intervals.
D. List the absolute values in order Solution:
$3,4.15,5.75,6.25,12$
E. Are your orders B and D the same? Why or why not?

Solution:
NO.
$B$ is using order and $D$ is dealing with absolute value
3. Christopher said, "I put these in order."

$$
-2.3,4.75,-8.2,12,-14.25
$$

Beth disagrees with Christopher. She knows he is wrong. Write a statement the Beth might say to show Christopher his mistake?

Solution:
Chris used the absolute value to put these in order. He was looking at the distance these numbers would be from zero.
-14.25, -8.2, -2.3, 4.75, 12
4. Michelle's debt is $\$ 25$ and Sharif's debt is $\$ 45$. Both students think they have the most money. Who is correct? Write a statement to PROVE who has the most.

Solution:
Michelle is correct
Michelle's value is -25 and Sharif's value is -45
-25 is a larger number than -45

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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Absolute Value and Ordering

For negative numbers, as the absolute value increases the value of the number decreases. The greater the debt the less money you have. The greater the depth the farther away from the surface.
1.
A. A mother dolphin is 150.25 meters below sea level. Her calf is 45 meters below sea level. Which dolphin is farthest from the surface?
B. Draw this situation on a vertical number line? Does this illustrate absolute value or ordering (for ordering use $<,>$ )? Justify your answer.
C. A mother whale is at 35 meters below the surface and her calf is at the surface. How far does the calf have to swim to get to its mother?
D. Draw this situation on a vertical number line? Does this illustrate absolute value or ordering (for ordering use $<,>$ )? Justify your answer.
2. Tonya, Beverly, Ashley, Janet, and Anne want to figure out who has the most debt and who has the most money.

> Tonya -\$5.75
> Beverly $\$ 12$
> Janet $\$ 6.25$
> Anne -\$4.15
> Ashley -\$3
A. Graph the five numbers on the number line.
B. Put the numbers in order.

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
C. What is the absolute value of each term in the list? Graph the absolute value of each term on a horizontal number line.
D. List the absolute values in order
E. Are your orders B and D the same? Why or why not?
3. Christopher said, "I put these in order."

$$
-2.3,4.75,-8.2,12,-14.25
$$

Beth disagrees with Christopher. She knows he is wrong. Write a statement the Beth might say to show Christopher his mistake?
4. Michelle's debt is $\$ 25$ and Sharif's debt is $\$ 45$. Both students think they have the most money. Who is correct? Write a statement to PROVE who has the most.

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

Version 2 (See Scaffolded Task for all answers and comments)

## Absolute Value and Ordering

For negative numbers, as the absolute value increases, the value of the number decreases. The greater the debt, the less money you have. The greater the depth, the farther away the location is from the surface.

## Task Problem 1:

A mother dolphin is 150.25 meters below sea level. Her calf is 45 meters below sea level. Which dolphin is farthest from the surface? A mother whale is at 35 meters below the surface and her calf is at the surface. How far does the calf have to swim to get to its mother? Which statement deals with absolute value? Which statement deals with ordering? Justify your answer. Include words, number lines, models and/or illustrations.
Students will illustrate/model the two scenarios described in the problem and demonstrate their understanding of distance from zero (absolute value) and order through their explanations.

See teacher's edition in scaffolded version.

## Task Problem 2:

Tonya, Beverly, Ashley, Janet, and Anne look at their bank statements and find the balances listed below.

$$
\begin{aligned}
& \text { Tonya -\$5.75 } \\
& \text { Beverly- } \$ 12 \\
& \text { Janet -\$6.25 } \\
& \text { Anne -\$4.15 } \\
& \text { Ashley -\$3 }
\end{aligned}
$$

Who has the most money and who has the least debt? Is it the same person or two different people? Use words, number lines, models and/or illustrations to explain your answer. Students will illustrate/model the scenario described in the problem and demonstrate their understanding that as negative numbers increase (to the left on the number line), the value of the number decreases. Therefore, the person with the least debt has the most money.

## Task Problem 3:

Christopher said, "I put these in order."

$$
-2.3,4.75,-8.2,12,-14.25
$$

Beth disagrees with Christopher. She believes he is wrong. Is Beth or Christopher correct? Write a statement to show who is correct, including an explanation of the error made.
See teacher's edition in scaffolded version.

## Problem Task 4:

Michelle's debt is $\$ 25$ and Sharif's debt is $\$ 45$. Both students think they have the most money. Who is correct? Prove who has the least debt. See teacher's edition in scaffolded version.

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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Absolute Value and Ordering

For negative numbers, as the absolute value increases, the value of the number decreases. The greater the debt, the less money you have. The greater the depth, the farther away the location is from the surface.

## Task Problem 1:

A mother dolphin is 150.25 meters below sea level. Her calf is 45 meters below sea level. Which dolphin is farthest from the surface? A mother whale is at 35 meters below the surface and her calf is at the surface. How far does the calf have to swim to get to its mother? Which statement deals with absolute value? Which statement deals with ordering? Justify your answer. Include words, number lines, models and/or illustrations.

## Task Problem 2:

Tonya, Beverly, Ashley, Janet, and Anne look at their bank statements and find the balances listed below.

> Tonya -\$5.75
> Beverly- $\$ 12$
> Janet $-\$ 6.25$
> Anne -\$4.15
> Ashley -\$3

Who has the most money and who has the least debt? Is it the same person or two different people? Use words, number lines, models and/or illustrations to explain your answer.

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Task Problem 3

Christopher said, "I put these in order."

$$
-2.3,4.75,-8.2,12,-14.25
$$

Beth disagrees with Christopher. She believes he is wrong. Is Beth or Christopher correct? Write a statement to show who is correct, including an explanation of the error made.

## Task Problem 4

Michelle's debt is $\$ 25$ and Sharif's debt is $\$ 45$. Both students think they have the most money. Who is correct? Prove who has the least debt.

In this task, students will explore the meaning of absolute value in real-world context.

## STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
MGSE6.NS. 7 Understand ordering and absolute value of rational numbers.
MGSE6NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
MGSE6.NS.7b Write, interpret, and explain statements of order for rational numbers in realworld contexts.
MGSE6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a realworld situation.
MGSE6.NS.7d Distinguish comparisons of absolute value from statements about order.

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of problems involving points and distances in the coordinate plane.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about location and distance with visual representations.
3. Construct viable arguments and critique the reasoning of others. Students construct and critiques arguments about the practical application of the mathematics embedded in this task.
4. Model with mathematics. Students use a coordinate plane to calculate location and distance between points of interest.
5. Attend to precision. Students attend to the language of real-world situations to determine if positive or negative quantities/distances are being represented.
6. Look for and make use of structure. Students relate the structure of number lines to values of rational numbers as they use the coordinate plane.

## ESSENTIAL QUESTIONS

- How do I use absolute value in real-world context?
- Why is it useful for me to know the absolute value of a number?


## MATERIALS

- Copy of map for each group/person


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## TEACHER NOTES

In this task, students will be given the task of developing the itinerary for a Walking Field Trip in Downtown Atlanta (Act 1), then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions (Act 2). Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them. Once students have may their discoveries, it is time for the great reveal (Act 3). Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

Since students have not explored the Pythagorean Theorem or other advanced properties of triangles, they are not expected to calculate the direct, or closest distance between two points on the coordinate plane. Instead, have them figure the distance using only horizontal and vertical movement on the coordinate plane.

## TASK COMMENTS

While students will probably come up with a wide variety of questions, this task, however, is designed to promote a deeper understanding of absolute value in real-world context.

## More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

## ACT 1:

View map of the field trip area: (The student handout version is on a grid with the origin being the drop off point.)


Options to visit:

- Centennial Park
- Georgia Aquarium
- The World of Coke
- CNN Center
- State Bar of Georgia


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## ACT 2:

Student work time to plan a field trip in the area provided. What are the coordinates of the sites they would like to visit? What is the distance between the sites using the coordinates?

## ACT 3

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

Field Trip to Downtown Atlanta


Task Title: $\qquad$ Name:
$\qquad$
ACT 1
What did/do you notice?
$\square$
Main Question: $\qquad$
Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line


Low estimate
Place an " $x$ " where your estimate belongs
High estimate

## ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc...)

If possible, give a better estimate using this information: $\qquad$

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## Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

What was the result?

| Which Standards for Mathematical Practice did you use? |  |
| :--- | :--- |
| $\square$ Make sense of problems \& persevere in <br> solving them | $\square$ Use appropriate tools strategically. |
| $\square$ Reason abstractly \& quantitatively | $\square$ Attend to precision. |
| $\square$ Construct viable arguments \& critique the <br> reasoning of others. | $\square$ Look for and make use of structure. |
| $\square$ Model with mathematics. | $\square$ Look for and express regularity in repeated <br> reasoning. |

In this task, students plan the placement of the percussion of a marching band on a coordinate grid.

## STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
MGSE6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3)=3$, and that 0 is its own opposite.
MGSE6.NS.6b Understand signs of number in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
MGSE6.NS. 7 Understand ordering and absolute value or rational numbers.
MGSE6NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
MGSE6.NS.7b Write, interpret, and explain statements of order for rational numbers in realworld contexts.
MGSE6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a realworld situation.
MGSE6.NS.7d Distinguish comparisons of absolute value from statements about order. MGSE6.NS. 8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
MGSE6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply those techniques in the context of solving real-world mathematical problems.

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of problems involving points and polygons in the coordinate plane.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about rational numbers with their visual representations.

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3. Construct viable arguments and critique the reasoning of others. Students construct and critiques arguments regarding number line and coordinate plane representations.
4. Model with mathematics. Students use number lines and coordinate planes to compare numbers in real-world contexts.
5. Use appropriate tools strategically. Students select and use tools such as two-color counters, number line models and the coordinate plane to represent situations involving positive and negative numbers.
6. Attend to precision. Students attend to the language of real-world situations to determine if positive or negative quantities/distances are being represented.
7. Look for and make use of structure. Students relate the structure of number lines to values of rational numbers as they use the coordinate plane.
8. Look for and express regularity in repeated reasoning. Students relate new experiences to experiences with similar contexts when studying positive and negative representations of distance and quantity. In the study of absolute value, students demonstrate repeated reasoning by showing that both positive and negative quantities represent the same distance from zero

## ESSENTIAL QUESTIONS

- Why is it useful for me to know the absolute value of a number?
- Why is graphing on the coordinate plane helpful?
- How do I use positive and negative numbers to represent quantities in real-world contexts?
- How do statements of inequality help me place numbers on a number line?


## MATERIALS

Paper
Pencil
Graph Paper

## TASK COMMENTS

Students justify thinking use distance, absolute value, opposites, number lines, coordinates, and positive and negative numbers. Students also revisit concepts learned earlier such as area and perimeter and computing fluently with multi-digit numbers using standard algorithms. Students must solve problems using real-world contexts. In addition, students must communicate mathematically to help justify solutions.

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Culminating Task: Sounds of the Band

## Part I: Planning

You are the child of a high school band director. Your dad knows that you are studying about the coordinate plane, absolute value, opposites, and positive and negative numbers. He is seeking your assistance with planning his next competition. Of course, you will be paid for your work so that you can purchase those items you have been wanting.

Below is a coordinate graph. The area that will be used is 36 feet wide by 36 feet long. The graph is used to help plan the location of the band members. One of the requirements of the competition is that members of the percussion section take the same number of steps from the origin to their position. In addition, each point in the coordinate plane has a matching point when reflected over the x and y axes in order to create a visually pleasing, symmetrical program. The final requirement is that diagonal moves may not be used.

1. There are ten members of the percussion section. Create a visual representation of where the band members will be placed on the field.

## Solution

Answers will vary;
Students should place ten members on the coordinate grid. Those members should be placed in four different quadrants and each quadrant must be symmetrical to the others.


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2. Label the coordinates for the location of each of the band members. How far from the origin is each band member?

## Solution

Answers will vary.
(2, 4); (3, 3); (0, 6); (2, -4); (-2, -4); (0, -6); (-3, 3); (-2, 4)
In the example below, all points are 6 units ( 18 feet) from the origin.
3. What patterns or relationships do you see among the coordinates?

## Solution

Answers will vary.
Students should discuss distance from the origin and ordered pairs as reflections across the $x$ axis and $y$-axis. Students should also comment about the relationships between the $x$ and $y$ coordinates (Ex: For a reflection over the $x$-axis: The $x$ coordinate stays the same. The $y$ coordinate becomes the opposite, and for a reflection over the $y$-axis: The x coordinate becomes the opposite. The y coordinate stays the same)

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During the second part of the competition, two schools bring their percussion sections together for a performance. There are now sixteen members. Your job is to create identical quadrilaterals using four members in each quadrant. The coordinate plane must be symmetrical when folded on the x axis and the y axis.

This time, the child of the other band director is helping. That child has already placed four of the band members.
4. Place the remaining members of the band.

## Solution

See coordinate plane.

5. What is the area of each quadrilateral?

## Solution

The area of each quadrilateral can be found by using the formula $A=l w$. The length of each rectangle is 12 feet and the width of each rectangle is 9 feet. Therefore, the area is 108 square feet.

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
6. What is the perimeter of each quadrilateral?

## Solution

The perimeter of each quadrilateral can be found by using the formula $\boldsymbol{P}=2 l+2 w$ or $\boldsymbol{P}=I+$ $l+w+w$. The length of each rectangle is 12 feet and the width of each rectangle is 9 feet. Therefore, the perimeter of the rectangle is 42 feet.
7. How do you know that each quadrilateral is identical?

## Solution

Answers will vary.
Students can discuss the fact that the area and perimeter are the same. They can tell that each rectangle was created by reflecting one of the other rectangles across the $x$ and $y$-axis.
8. Justify that each quadrilateral is a "mirror image" of the other quadrilaterals.

## Solution

Answers will vary.
Student justification should include ideas such as distances between points and ideas about reflections of points (Ex: For a reflection over the x-axis: The x coordinate stays the same. The $y$ coordinate becomes the opposite, and for a reflection over the $y$-axis: The $x$ coordinate becomes the opposite. The y coordinate stays the same)
9. Look at the quadrilateral in Quadrant I. What are the locations of the band members?

## Solution

$(2,1),(5,1),(5,5)$, and $(2,5)$
10. What are the locations of the band members in Quadrant IV? What is the relationship between the coordinates of the band members in Quadrant IV as related to the coordinates of band members Quadrant I?

## Solution

The locations of the band members in Quadrant IV are (2, -1), (5, -1), (5, -5), and (2, -5). Because figure the figure in quadrant IV was created by flipping the figure in Quadrant I over the $x$-axis, the $x$ coordinate stays the same. The $y$ coordinate becomes the opposite.
11. What are the locations of the band members in Quadrant II? What is the relationship between the coordinates of the band members in Quadrant II as related to the coordinates of band members in Quadrant I?

## Solution

The locations of the band members in Quadrant II are (-2, 1), (-5, 1), (-5, 5), and (-2, 5).
Because figure the figure in quadrant II was created by flipping the figure in Quadrant I over the $y$-axis, the $x$ coordinate becomes the opposite. The y coordinate stays the same.

During the third part of the competition, your mom, a sixth grade mathematics teacher, decides to help your dad place the members of the percussion section. She places the 12 band members that you see below.

12. What is the area of each triangle?

## Solution

The area of a triangle can be found by using the formula $A=\frac{1}{2} b h$ or $A=\frac{b h}{2}$. The base of the triangle is 9 feet and the height of the triangle is 12 feet. Therefore, the area of the triangle is 54 square feet.
13. What is the distance of the points $(3,5)$ and $(3,-5)$ from the $x$-axis? What is the distance of those two points from each other?

## Solution

The distance of $(3,5)$ from the $x$-axis is 5 units. The distance of $(3,-5)$ from the $x$-axis is 5 units. The distance of those two points from each other is 10 units.
14. What is the distance of the points $(-6,5)$ and $(6,5)$ from the $y$-axis? What is the distance of those two points from each other?

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Solution

The distance of $(-6,5)$ from the $y$-axis is 6 units. The distance of $(6,5)$ from the $y$-axis is 5 units. The distance of those two points from each other is 10 units.
15. Given the point $(6,-1)$, what are the coordinates if the point is reflected across the $x$-axis? Given the point $(6,-1)$, what are the coordinates if the point is reflected across the $y$-axis?

## Solution

Given the point (6, -1 ), the point that has been reflected across the $x$-axis is $(6,1)$. The point that has been reflected across the $y$-axis is (-6, -1).
16. Given the point $(-3,5)$, what are the coordinates if the point is reflected across the x-axis? Given the point $(-3,-5)$, what are the coordinates if the point is reflected across the y-axis?

## Solution

Given the point $(-3,5)$, the point that has been reflected across the $x$-axis is $(-3,-5)$. The point that has been reflected across the $y$-axis is $(3,5)$.
17. If all students start at the origin, which student or students walk the greatest distance to the assigned location? Remembering that students cannot walk diagonally, give the location of those students as an ordered pair. How far did those students walk? Write a number sentence using absolute value to prove your answer.

## Solution

The students that walk the greatest distance are those students located at (-3, 5), (3, 5), (3, -5), and $(-3,-5)$. All of the above students walked a distance of 24 feet. $|9|+|15|=24$.
18. Which point, $(3,-1)$ or $(3,-5)$, is a greater distance from the $x$-axis? Prove your answer using a statement of inequality.

## Solution

The point $(3,-5)$ is a greater distance from the $x$-axis. $|-1|<|-5|$ or $|-5|>|-1|$.
19. Which point, $(-3,-1)$ or $(-6,1)$, is a greater distance from the $y$-axis? Prove your answer using a statement of inequality.

## Solution

The point (-6, -1) is a greater distance from the $y$-axis. $|-3|<|-6|$ or $|-6|>|-3|$.

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## Part II: Pay Day

Your dad is now going to pay you for your help. You helped your dad for a total of five hours. He is paying you $\$ 4.50$ for each hour that you helped him.

1. How much money does he give you?

Solution
You earn $\$ 4.50$ multiplied by 5 hours for a total of $\$ 22.50$.
2. If you have $\$ 32.04$ already saved, how much money do you have now?

Solution
You have \$32.04 plus \$22.50 for a total of \$54.54.
3. You want to purchase a shirt that will cost $\$ 54.98$ with tax. How much money do you need to save?

Solution
You need \$0.44 to buy the shirt (\$54.98 - \$54.54).

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7

## CULMINATING TASK: SOUNDS OF THE BAND

## Part I: Planning

You are the child of a high school band director. You dad knows that you are studying about the coordinate plane, absolute value, opposites, and positive and negative numbers. He is seeking your assistance with planning his next competition. Of course, you will be paid for your work so that you can purchase those items you have been wanting.

Below is a coordinate graph. The area that will be used is 36 feet wide by 36 feet long. The graph is used to help plan the location of the band members. One of the requirements of the competition is that members of the percussion section take the same number of steps from the origin to their position. In addition, each point in the coordinate plane has a matching point when reflected over the $x$ and $y$ axes in order to create a visually pleasing, symmetrical program. The final requirement is that diagonal moves may not be used.

1. There are ten members of the percussion section. Create a visual representation of where the band members will be placed on the field.

2. Label the coordinates for the location of each of the band members. How far from the origin is each band member?

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Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
3. What patterns or relationships do you see among the coordinates?

During the second part of the competition, two schools bring their percussion sections together for a performance. There are now sixteen members. Your job is to create identical quadrilaterals using four members in each quadrant. The coordinate plane must be symmetrical when folded on the x axis and the y axis.

This time, the child of the other band director is helping. That child has already placed four of the band members.
4. Place the remaining members of the band.

5. What is the area of each quadrilateral?
6. What is the perimeter of each quadrilateral?
7. How do you know that each quadrilateral is identical?

# Georgia Department of Education 

Georgia Standards of Excellence Framework GSE Grade 6 Mathematics • Unit7
8. Justify that each quadrilateral is a "mirror image" of the other quadrilaterals.
9. Look at the quadrilateral in Quadrant I. What are the locations of the band members?
10. What are the locations of the band members in Quadrant IV? What is the relationship between the coordinates of the band members in Quadrant IV as related to the coordinates of band members Quadrant I?
11. What are the locations of the band members in Quadrant II? What is the relationship between the coordinates of the band members in Quadrant II as related to the coordinates of band members in Quadrant I?

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During the third part of the competition, your mom, a sixth grade mathematics teacher, decides to help your dad place the members of the percussion section. She places the 12 band members that you see below.

12. What is the area of each triangle?
13. What is the distance of the point $(3,5)$ and $(3,-5)$ from the $x$-axis? What is the distance of those two points from each other?
14. What is the distance of the point $(-6,5)$ and $(6,5)$ from the $y$-axis? What is the distance of those two points from each other?

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15. Given the point $(6,-1)$, what are the coordinates if the point is reflected across the x -axis? Given the point $(6,-1)$, what are the coordinates if the point is reflected across the $y$-axis?
16. Given the point $(-3,5)$, what are the coordinates if the point is reflected across the $x$-axis? Given the point $(-3,-5)$, what are the coordinates if the point is reflected across the $y$-axis?
17. If all students start at the origin, which student or students walk the greatest distance to the assigned location? Remembering that students cannot walk diagonally, give the location of those students as an ordered pair. How far did those students walk? Write a number sentence using absolute value to prove your answer.
18. Which point, $(3,-1)$ or $(3,-5)$, is a greater distance from the $x$-axis? Prove your answer using a statement of inequality.
19. Which point, $(-3,-1)$ or $(-6,-1)$, is a greater distance from the $y$-axis? Prove your answer using a statement of inequality.

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## Part II: Pay Day

Your dad is now going to pay you for your help. You helped your dad for a total of five hours. He is paying you $\$ 4.50$ for each hour that you helped him.

1. How much money does he give you?
2. If you have $\$ 32.04$ already saved, how much money do you have now?
3. You want to purchase a shirt that will cost $\$ 54.98$ with tax. How much money do you need to save?

## Apply and extend previous understandings of numbers to the system of rational numbers.

MGSE6.NS. 5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
https://www.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/277
https://www.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/278 http://figurethis.nctm.org/challenges/c46/challenge.htm

MGSE6.NS. 6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
https://www.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/1665
https://www.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/1998
https://www.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/1999
https://www.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/2009
http://www.shodor.org/interactivate/activities/GeneralCoordinates/
https://www.mathsisfun.com/data/cartesian-coordinates-interactive.html
http://www.explore.math.com/school/subject2/practice/S2U4L1/S2U4L1Pract.html
http://www.oswego.org/ocsd-web/games/BillyBug2/bug2.html
http://ccsstoolbox.agilemind.com/parcc/middle 1.html
MGSE6.NS.6a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3)=3$, and that 0 is its own opposite.

MGSE6.NS.6b Understand signs of number in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
http://mr-stadel.blogspot.com/2014/01/fun-with-dot-and-line.html
MGSE6.NS.6c Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

MGSE6.NS. 7 Understand ordering and absolute value of rational numbers. https://www.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/283

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https://www.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/284
https://www.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/285
https://www.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/286
https://www.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/288 http://ccsstoolbox.agilemind.com/parcc/middle_1.html

MGSE6NS.7a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
http://www.yummymath.com/2014/weather-extremes/
MGSE6.NS.7b Write, interpret, and explain statements of order for rational numbers in realworld contexts.

MGSE6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a realworld situation.

MGSE6.NS.7d Distinguish comparisons of absolute value from statements about order.
MGSE6.NS. 8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
$\underline{\text { https://www.illustrativemathematics.org/content-standards/6/NS/C/8/tasks/290 }}$

## Solve real-world and mathematical problems involving area, surface area, and volume.

MGSE6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply those techniques in the context of solving real-world mathematical problems.
https://www.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1188 https://www.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1997 http://illuminations.nctm.org/Lesson.aspx?id=1089


[^0]:    Mathematics • Grade 6 • Unit 7: Rational Explorations: Numbers and their Opposites
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[^1]:    STOP
    Note: Students should compare and order integers at this time.

